

# Errata

This document contains the most up to date set of known errors in *Closer and Closer: Introducing Real Analysis* by Carol S. Schumacher. There is a list of substantive errors at the beginning. A list of typographical errors is listed at the end of the document. If you find errors that are not on this list, I would be very grateful if you were to let me know about them. (Schumacherc@kenyon.edu.)

**Last Updated: March 22, 2019**

## 0.1 Sets and Notation

- Page 14—Indented portion of the first paragraph reads "Let  $\lambda$  be an arbitrary indexing set, . . ." It should, instead, read, "Let  $\Lambda$  be an arbitrary indexing set, . . ."

## 0.2 Functions

- Page 24—Problems 2(c) and 2(d) should read thus (I have underlined the words that need to be changed. The underline should not appear in the text.)
  - (c) If  $g \circ f$  is onto, then  $f$  is onto.
  - (d) If  $g \circ f$  is onto, then  $g$  is onto.

## 0.2 Sequences

- Page 34—In problem 1, the first line reads "Prove that the value of a term in an increasing sequence of natural numbers . . ." It should, instead, read, "Prove that the value of a term in a strictly increasing sequence of natural numbers . . ."
- 4.  $(s_i)$  is said to be **bounded from below** if there exists  $b \in A$  such that for all  $i \in \mathbb{N}$ ,  $b \leq s_i$ . In this case,  $b$  is called a **lower bound** for  $(s_i)$ .
- 5. If there exists  $c \in A$  such that for all  $i \in \mathbb{N}$ ,  $s_i \leq c$ , then we say that  $(s_i)$  is **bounded from above**. In this case,  $c$  is called an **upper bound** for  $(s_i)$ .

## 0.4 Sequences

- Page 14—Indented portion of the first paragraph reads "Let  $\lambda$  be an arbitrary indexing set, . . ." It should, instead, read, "Let  $\Lambda$  be an arbitrary indexing set, . . ."

## 1.4 Least Upper Bound Axiom

- Page 60—Problem 3 is false, as stated.  $t$  must be non-negative.

*Original phrasing:*

Let  $t \in \mathbb{R}$  and let  $S \subset \mathbb{R}$  that is bounded above.

*Suggested rephrasing:*

Let  $t \in \mathbb{R}^+$  and let  $S \subset \mathbb{R}$  that is bounded above.

- Page 60—In problem 4, the sets  $S$  and  $T$  need to be non-empty. The problem should say, “Let  $S$  and  $T$  be non-empty subsets of  $\mathbb{R}$  that are bounded above.”

## 2.2 The Euclidean Metric on $\mathbb{R}^n$

- Page 65—Middle of the page. The definition of the metric on  $\mathbb{R}^n$  is mistyped as a fraction. It should read:

$$d((a_1, a_2, \dots, a_n), (b_1, b_2, \dots, b_n)) = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + \dots + (a_n - b_n)^2}.$$

- Page 67 —At the end of Lemma 2.2.3: equality holds in the Cauchy-Schwarz inequality if and only if  $\mathbf{v}$  is a scalar multiple of  $\mathbf{u}$  or  $\mathbf{u}$  is the zero vector.
- Page 71—The footnote refers to Problem 5b, it should, instead refer to Problem 5a.
- Page 88—Proof of Theorem 3.4.3; top line. The sentence “So  $a - k < a - a_n$  for all  $\dots$ ” should instead say “So  $a - k \leq a - a_n$  for all  $\dots$ ”

## 3.4 Sequences in $\mathbb{R}$

- Page 92— In problem 10,  $\lim_{n \rightarrow \infty} c^{\frac{1}{n}} = 1$  should be  $\lim_{n \rightarrow \infty} c^{\frac{1}{n}} = 1$ .
- Page 93—In problem 11, the reference to Excursion D.4.7 should really be a reference to Exercise D.4.7.

## 3.7 Open Sets, Closed Sets, and the Closure of a Set

- Page 99—In part 5 of Exercise 3.7.4 the statement “Let  $x \in \overline{X}$ ” should, instead, be “Let  $x \in X$ .”

## 4.2 Limit of a Function at a Point

- Page 108—In Theorem 4.2.3 the word “approaches” should be “approaches.”
- Page 108—In the statement of Theorem 4.2.4. In the hypothesis of the theorem,  $a$  should be a limit point of  $X$ , not just any element of  $X$ .
- Page 109—in the proof of Theorem 4.2.4, first line of page 109. The line begins “. . . then  $d_Y(f(x), f(a)) < \epsilon$ .” This should, instead, read “. . . then  $d_Y(f(x), L) < \epsilon$ .”
- Page 109—in the proof of Theorem 4.2.4, the subheading for the second part should read  $\sim 1 \implies \sim 2$  rather than  $2 \longleftarrow \dots$ .
- Page 109—in the proof of Theorem 4.2.4, the last sentence reads “However,  $(x_n)$  *does* converge to  $a$ .”

The proof should, instead, continue as follows: “The sequence  $(x_n)$  must have either a sequence of distinct terms or a constant subsequence. Because  $(x_n) \rightarrow a$  and for all  $n \in \mathbb{N}$ ,  $x_n \neq a$ ,  $(x_n)$  cannot have a constant subsequence. It must therefore have a subsequence  $(x_{i_n})$  of distinct terms. This sequence converges to  $a$  and yet for all  $n \in \mathbb{N}$ ,  $d_Y(f(x_{i_n}), L) \geq \epsilon$ .”

### 4.3 Continuous Functions

- Page 112—in the proof of Theorem 4.3.5, second line from the top of the page. “But then if  $d_X(y, x) < \delta$ ,  $f(z) \in V \dots$ ” should, instead, read “But then if  $d_X(y, x) < \delta$ ,  $f(y) \in V \dots$ ”.
- Page 114—The point  $a$  referred to in problem 7 must be a limit point of  $X$ , otherwise the limit is not defined.

*Original phrasing:*

. . . Prove that  $f$  is continuous at  $a \in X$  if and only if . . .

*Should be:*

. . . Prove that  $f$  is continuous at a limit point  $a$  of  $X$  if and only if . . .

### 4.4 Uniform Continuity

- Page 116—For problem 6(c). Add the parenthetical statement

(Assume for now that the difference of two continuous, real-valued functions is continuous. This will be proved in Section 5.3.)

at the end of the statement of the problem.

## 5.1 Limits, Continuity, and Order

- Page 122—Second paragraph after **Some Useful Special Cases**. The sentence with the bad reference (??) should be: “Corollary 5.1.9 is a special case of Theorem 5.1.1.”
- Page 123—In corollary 5.1.13, the second bullet item should reads  $f(a) \leq h(a)$  and, instead, should read  $f(a) = h(a)$ .

### 5.3 Limits, Continuity, and Arithmetic

- Page 127—In theorem 5.3.1(4), the statement reads “Assume further that  $g(x) \neq 0$  on some interval containing  $a \dots$ ” it should, instead, be “Assume further that  $g(x) \neq 0$  on some open set containing  $a \dots$ ”

### 7.1 Compact Sets

- Page 143—In part (b) of problem 17. The set  $X$  should be

$$X = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1, x > 0 \text{ and } y > 0\}.$$

### 7.2 Continuity and Compactness

- Page 145—Problem 5; Though this is not, strictly speaking, an error, it would be better if the second hint said that it is easier to prove the theorem by contradiction. (Contrapositive is also possible using the same idea, but a bit more tricky.) Problem 11 at the end of Section 7.1 can also be useful.

### 8.1 Connected Sets

- Page 153—Problem 1. The problem reads,

Prove the IVT for a continuous function  $f : [a, b] \rightarrow \mathbb{R}$  as follows. Suppose that  $\gamma$  is between  $f(a)$  and  $f(b)$ . Let  $c = \sup\{x \in [a, b] : f(x) \leq \gamma\}$ . Show that  $f(c) = \gamma$ .

It should, instead, read

Prove the IVT for a continuous function  $f : [a, b] \rightarrow \mathbb{R}$  as follows. Suppose that  $f(a) \leq \gamma \leq f(b)$ . Let  $c = \sup\{x \in [a, b] : f(x) \leq \gamma\}$ . Show that  $f(c) = \gamma$ . Modify the argument for the case when  $f(b) \leq \gamma \leq f(a)$ .

### 9.2 The Derivative

- Page 163— In the first line of the second paragraph contained in the box, “ $y \rightarrow 0$ ” should be “ $y \rightarrow x$ .”

- Page 163—In the last line in the box “the expression” should be “the expression in Theorem 9.2.2.”

## 9.4 The Derivative

- Page 172—The statement of Rolle’s theorem should specify that  $a < b$ .
- Page 173— In the statement of Corollary 9.4.7, the forward direction requires that the interval  $I$  be an open interval. It can be useful to note, however, that the converse does not. Alternatively, one may assume that the function is differentiable on  $I$ .
- Page 173—In the statement of Corollary 9.4.8, the functions  $f$  and  $g$  need to be differentiable functions.

## 9.7 Polynomial Approximation and Taylor’s Theorem

- Page 184—The hypothesis of this theorem is inadequate to handle the case when either  $t$  or  $s$  is an endpoint of the interval  $[a, b]$ . To make the theorem fully correct, the hypothesis should be that  $f$  has a continuous derivative on  $[a, b]$  and is twice differentiable on  $(a, b)$ .
- Page 184—In the proof of Theorem 9.7.1, third line from the end. The line reads: “But  $A''(x) = f'' - M$ , so  $A''(c) = 0$  which implies that  $M = f''(c)$ ” It should, instead, read “But  $A''(x) = f'' - M$ . So  $A''(c) = 0$  implies that  $M = f''(c)$ ”

## 10.1 Iteration and Fixed Points

- Footnote on page 196. In the piecewise defined function,  $b$  should, instead, be  $x$ .
- Page 196—Though not an error, as such, the  $\supset$  in Problem 5 may be misleading to students who work hard to use the fact that the inclusion is proper. Changing it to  $\supseteq$  does not affect the argument at all. (The case when  $f([a, b]) = [a, b]$  is “covered” by problem 4.)
- Page 197—Problem 8, second line reads “...  $1 \leq k \leq 3$  ...” It should, instead, read “...  $1 < k < 3$ .”

## 11.4 Families of Riemann Sums

- Page 223—Last line of the page reads:

$$N^*(z_j - z_{i-1}) \text{ where } N^* = \sup\{f(x) : x \in [z_i, z_j]\}.$$

It should read, instead,

$$N^*(z_j - z_i) \text{ where } N^* = \sup\{f(x) : x \in [z_i, z_j]\}.$$

## 11.5 Existence of the Integral

- Page 228—The definition of the function in Example 11.5.3 needs to be modified slightly. Add “or 0” in the first line of the definition:

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is irrational or } 0 \\ \frac{1}{q} & \text{if } x \text{ is rational and } x = \frac{p}{q} \end{cases}$$

- Page 228—Theorem 11.5.4. The third sentence reads “If  $f : K \rightarrow \mathbb{R}$  is a function, then the integral  $\int_a^b f \dots$ .” It should, instead, read “If  $f : K \rightarrow \mathbb{R}$  is a function and  $a < c < b$ , then the integral  $\int_a^b f \dots$ .”
- Page 232—The expression at the bottom of the page reads.

$$\left| \mathcal{R}(f, P) - \left( \int_a^c f + \int_c^b f \right) \right|.$$

It should read, instead,

$$\left| \mathcal{R}(f, P) - \left( \int_a^c f + \int_c^b f \right) \right|.$$

- Page 233—Problem 4(c) currently reads: “Now use the result from parts (a) and (b) to remove the restriction that  $a < b < c$ . (You may need to break this into several cases.)” It should, instead say “Assume any two of the three integrals  $\int_a^c f$ ,  $\int_c^b f$  and  $\int_a^b f$  exist. Use the result from parts (a) and (b) to remove the restriction that  $a < c < b$ . (You may need to break this into several cases.)”
- Page 233—The definition of the function in Problem 5 needs to be modified slightly. Add “or 0” in the first line of the definition:

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is irrational or } 0 \\ \frac{1}{q} & \text{if } x \text{ is rational and } x = \frac{p}{q} \end{cases}$$

And the word “non-zero” needs to be added to the statement in part (a): “Show that  $f$  is discontinuous at every non-zero rational number.”

## 11.6 The Fundamental Theorem of Calculus

- Page 234—The first part of theorem 11.6 is false, as stated. With the current hypothesis, the area accumulation function can only be shown to be uniformly continuous on any closed subinterval of the interval  $I$ . There are various ways to rephrase the theorem: If  $f$  is assumed to be bounded

on  $I$ , then the conclusion is correct, as stated. If  $I$  is assumed to be a closed interval, then the conclusion is correct, as stated.

The second part of the theorem is correct without any change in the hypotheses, since one can restrict attention to a closed subinterval of  $I$  that contains  $c$  and the proof goes through without any problem.

## 12.2 Uniform Convergence

- Page 244—In problem 10. The sequence of functions need not be continuous. (While this is not precisely an error, it may be confusing for students.)
- Page 245—In problem 12, the next to the last line before part (a). “... that  $\lim_{m \rightarrow \infty} = f(x)$ ” should, instead, read “... that  $\lim_{m \rightarrow \infty} f(x_m) = f(x)$ .”

## 12.3 Series of Functions

- Page 247—Lemma 12.3.3 reads “... there exists  $N \in \mathbb{N}$  such that for all  $m > n > N \dots$ ” It should, instead read, “... there exists  $N \in \mathbb{N}$  such that for all  $m \geq n > N \dots$ ”

## 13.1 What Are We Studying

- Page 259—Theorem 13.1.2(1). The second line reads “...  $\lim_{k \rightarrow \infty} \mathbf{f}(\mathbf{y}_n) = \mathbf{b}$  if and only if for each  $i = 1, 2, \dots, m$ ,  $\lim_{k \rightarrow \infty} f_i(\mathbf{y}_n) = b_i$ .” the subscripts  $n$  should, instead be  $k$ 's. The line should read “...  $\lim_{k \rightarrow \infty} \mathbf{f}(\mathbf{y}_k) = \mathbf{b}$  if and only if for each  $i = 1, 2, \dots, m$ ,  $\lim_{k \rightarrow \infty} f_i(\mathbf{y}_k) = b_i$ .”

## 13.2 Thinking Intuitively

- Page 260—6 lines from the bottom of the page; the function should be a  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$  instead of  $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$ .

## 13.3 Analysis in Linear Spaces

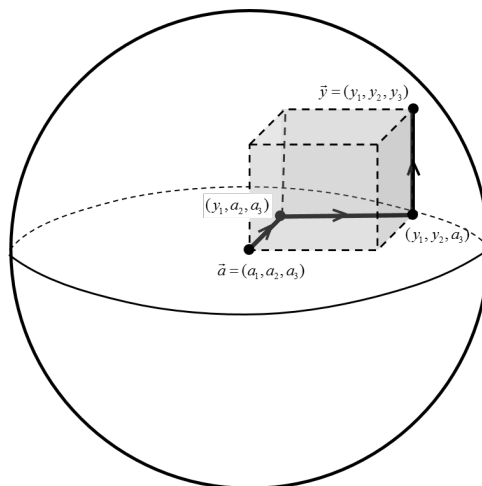
- Page 263—items 5 and 6 in the list at the top should begin “For all scalars  $s$  and  $r \in \mathbb{R} \dots$ ”
- Page 263—In the definition of linearly independent, the final line should read “... if and only if  $r_1 = r_2 = \dots = r_n = 0$ .”

- Page 263—In Exercise 13.3.3, the set describing the basis has the wrong number of elements. The first line should read “Suppose that  $\{\mathbf{b}_i\}_{i=1}^n$  is a basis for  $\mathbb{R}^n$ ” instead of “Suppose that  $\{\mathbf{b}_i\}_{i=1}^k$  is a basis for  $\mathbb{R}^n$ .”
- Page 265—Theorem 13.3.9(3). The linear transformation referred to in this part of the problem needs to be one-to-one. The problem should read: “Suppose that  $\mathbf{L}$  is one-to-one, then the set  $\{\mathbf{L}(\mathbf{e}_i)\}_{i=1}^n$ , the image of the standard basis, is linearly independent in  $\mathbb{R}^m$ .”
- Page 265 — In Theorem 13.3.11, the  $L$  in the last line should be bold-faced: “. . . given by the vectors  $\{\mathbf{L}(\mathbf{e}_i)\}_{i=1}^n$ .”
- Page 267-268—In Theorem 13.3.20. Because  $n$  is the dimension of the space in this problem, every subscript  $n$  should be changed to an  $i$ . The theorem should read:  
Let  $(\mathbf{x}_i)$  and  $(\mathbf{y}_i)$  be sequences in  $\mathbb{R}^n$  converging to  $\mathbf{x}$  and  $\mathbf{y}$ , respectively. Let  $(t_i)$  be a sequence in  $\mathbb{R}$  that converges to a scalar  $t$ . Let  $k$  be an arbitrary scalar. Then the following facts hold:
  1.  $(k\mathbf{x}_i)$  converges to  $k\mathbf{x}$ .
  2.  $(t_i\mathbf{x}_i)$  converges to  $t\mathbf{x}$ .
  3.  $(\mathbf{x}_i + \mathbf{y}_i)$  converges to  $\mathbf{x} + \mathbf{y}$ .
  4.  $(\mathbf{x}_i \cdot \mathbf{y}_i)$  converges to  $\mathbf{x} \cdot \mathbf{y}$ .
  5.  $\|\mathbf{x}_i\| \rightarrow \|\mathbf{x}\|$
- Page 268— In Definition 13.3.21, change  $\mathbf{T}$  to  $\mathbf{L}$  in the second line.
- Page 270—In Problem 10—Because  $n$  is the dimension of the space in this problem, every subscript  $n$  should be changed to an  $i$ . The problem should read  
Let  $(\mathbf{x}_i)$  be a sequence in  $\mathbb{R}^n$  and let  $(t_i)$  be a sequence of scalars.
  - (a) Suppose that  $(\mathbf{x}_i)$  converges to  $\mathbf{0}$  and that  $(t_i)$  is bounded in  $\mathbb{R}$ . Prove that  $(t_i\mathbf{x}_i)$  converges to  $\mathbf{0}$ .
  - (b) Suppose that  $(t_i)$  is a sequence in  $\mathbb{R}$  that converges to 0, and  $(\mathbf{x}_i)$  is a bounded sequence in  $\mathbb{R}^n$ . Prove that  $(t_i\mathbf{x}_i)$  converges to  $\mathbf{0}$ .
- Page 271—Problem 12, first line—“established in Lemma 13.3.21 . . .” should, instead, read, “. . . established in Corollary 13.3.23.”
- Page 271—In problem 15(a). There is a typographical error in the description of  $B_r(\mathbf{x})$ . It reads “ $\mathbf{x} + s\mathbf{u} : 0 \leq s \leq r \dots$ ” It should, instead, read “ $\mathbf{x} + s\mathbf{u} : 0 \leq s < r \dots$ ”
- Page 271—In problem 16, second line: it reads “. . . if and only if  $\mathbf{L}(\mathbf{e}_i) = \mathbf{S}(\mathbf{e}_i) \dots$ ” It should, instead, read “. . . if and only if  $\mathbf{T}(\mathbf{e}_i) = \mathbf{S}(\mathbf{e}_i) \dots$ ”



## 13.4 Local Linear Approximation

- Page 272-273—in Exercise 12.4.2, while this is not technically an error, it may be helpful to change  $\mathbf{r}$  to  $\mathbf{e}$  so that students do not confuse the error term in Definition 13.4.1 with this one. They are, of course related, but they are different functions.
- Page 277—In Theorem 13.4.8—The second and third lines should read: “... field. Let  $\mathbf{a} \in \mathbf{E}$ . Suppose that  $\mathbf{f}$  is differentiable at  $\mathbf{a}$ . Then all the partial derivatives of  $\mathbf{f}$  exist at  $\mathbf{a}$ .  
The last line before the equation at the end reads “it follows that for  $i = 1, 2, \dots, n$  and  $j = 1, 2, \dots, m, \dots$ ” The last quantification is unnecessary. It should read “it follows that for  $i = 1, 2, \dots, n, \dots$ ”
- Page 278— Paragraph 2, second line. The function  $f$  should be bold-faced as it is in the first line.
- Page 279—In Theorem 13.4.12, in “Step 1.” The very beginning reads “Define  $r : E \rightarrow \mathbb{R} \dots$ ” It should, instead, say “Define  $e : E \rightarrow \mathbb{R} \dots$ ”
- Page 280—in the picture. The bolded route starting at  $\mathbf{a} = (a_1, a_2, a_3)$  should go *back* to the bottom, back left corner, then *right* to the bottom, back, right corner, then *up* to the back, right, upper corner, ( $\mathbf{y} = (y_1, y_2, y_3)$ ). The correct picture looks like this.



- Page 281—In the middle of Step 5 a displayed equation reads

$$g'_i(c_i)(y_i - a_i) = g(y_i) - g(a_i).$$

the functions  $g$  on the right hand side should, instead be  $g_i$ 's. It should read:

$$g'_i(c_i)(y_i - a_i) = g_i(y_i) - g_i(a_i).$$

- Page 282—In problem 3, in the first line:  $f : E \rightarrow \mathbb{R}^m$  should read  $\mathbf{f} : E \rightarrow \mathbb{R}^m$ .
- Page 283—Problem 7. The sequence suggested in the hint should be an *arbitrary* sequence of real numbers that goes to zero. Not a *decreasing sequence of positive* real numbers that goes to zero. Moreover, using  $n$  as a subscript here is problematic. The hint should read: “*Hint*: Start with the expression guaranteed by the differentiability of  $\mathbf{f}$  at  $\mathbf{a}$ . Fix  $i \leq n$ . Let  $(t_k)$  be a sequence of real numbers that goes to zero. consider the sequence  $\mathbf{y}_k = \mathbf{a} + t_k \mathbf{e}_i$ .”
- Page 283—Problem 8. The displayed equation at the very end should read,

$$f(\mathbf{x}) = \nabla f(\mathbf{a}) \cdot (\mathbf{x} - \mathbf{a}) + f(\mathbf{a}) + r(\mathbf{x}), \quad \text{and} \quad \lim_{\mathbf{x} \rightarrow \mathbf{a}} \frac{|r(\mathbf{x})|}{\|\mathbf{x} - \mathbf{a}\|} = 0.$$

- Page 284—Problem 13. The function is incorrect. It should, instead, be

$$f(x, y) = \begin{cases} \frac{yx + 2xy^2}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

- Page 286—The displayed expression near the bottom of the page is off by a minus sign.

$$D_1 D_2 f(\mathbf{a}) \approx \frac{\frac{f(a_1, a_2+k) - f(a_1, a_2)}{k} - \frac{f(a_1+h, a_2+k) - f(a_1+h, a_2)}{k}}{h}$$

should, instead, be

$$D_1 D_2 f(\mathbf{a}) \approx \frac{\frac{f(a_1+h, a_2+k) - f(a_1+h, a_2)}{k} - \frac{f(a_1, a_2+k) - f(a_1, a_2)}{k}}{h}.$$

### 13.5 The Mean Value Theorem for Functions of Several Variables

- Page 290— the function is a vector-valued function and so should be bold-faced. In the first line, the second sentence should read: “Let  $\mathbf{f} : E \rightarrow \mathbb{R}^m$  be a ...” In the third line, the first full sentence should start: “If  $\mathbf{f}$  is differentiable ...”

### Excursion C.2 Exponents

- Page 305—In Theorem C.2.10, the reference to the positive real number  $b$  needs to be removed. The theorem should read: “Let  $a$  be a positive real number. Let  $r$  and  $s$  ...”

## Excursion D.4 Some Important Special Sequences

- Page 317—Middle of the page (Step 2. in the proof sketch for Theorem D.4.5). “Excursion 3” should instead be “Excursion C.”

## Excursion F.1 Double Sequences and Convergence

- Page 328—In definition F.1.6, 4<sup>th</sup> line reads, “. . .  $N \in \mathbb{N}$  such that for all  $m > N$  and all  $N \in \mathbb{N}$  . . . .” It should, instead, read “. . .  $N \in \mathbb{N}$  such that for all  $m > N$  and all  $n \in \mathbb{N}$  . . . .”
- Page 329—The metric space in Theorem F.1.7 needs to be complete. In other words, the hypothesis should read “Let  $X$  be a complete metric space, . . . .”

## Excursion H.1 Series of Real Numbers

- Page 336—Theorem H.1.5 in the last line before the displayed inequality, “ $n > m > N$ ” should, instead, read “ $n \geq m > N$ .”

## Excursion H.4 Rearranging the Terms of a Series

- Page 353—As stated in Lemma H.4.5, the last word in problem 2 should be “diverge” not “converge.”

## Excursion I.1 Regular Riemann Sums

- Page 359—The second displayed expression reads

$$\sum_{i=0}^{n-1} f(x_i)(x_i - x_{i-1}).$$

It should, instead, be

$$\sum_{i=1}^n f(x_{i-1})(x_i - x_{i-1})$$

## Excursion J.1 Power Series

- Page 366—The second and fourth power series given in Exercise J.1.6 should start at  $n = 1$ :

$$\sum_{n=0}^{\infty} \frac{3}{n^2} (x-5)^n \text{ should instead be } \sum_{n=1}^{\infty} \frac{3}{n^2} (x-5)^n.$$

$$\sum_{n=0}^{\infty} \frac{1}{n} (x-5)^n \text{ should instead be } \sum_{n=1}^{\infty} \frac{1}{n} (x-5)^n.$$

### Excursion K Everywhere Continuous, Nowhere Differentiable

- Page 379—The lower estimate in step 5 of the outline reads  $\frac{1}{2}(3^m - 1)$ . It should, instead, be  $\frac{1}{2}(3^m + 1)$

### M.1 Solving Systems of Equations

- Page 392—In Example M.1.3, the second line of text should read: “Suppose we define  $h : \mathbb{R}^2 \rightarrow \mathbb{R}$  by  $h(x, y) = \cos(x - y) - \sin(xy) \dots$ ”

### M.2 The Implicit Function Theorem

- Page 396—The second line in the paragraph immediately below the figure should read: “...line to the circle at  $(1, 0)$  is *vertical*...”
- Page 400—In Figure M.5, the left disk/ball is in  $\mathbb{R}^m$  not in  $\mathbb{R}^n$ .

### Excursion M.4 The Inverse Function Theorem

- Page 407—in problem 2, the second line, “The reverse is also possible” should instead read “The reverse is also possible provided that we assume all of the partial derivatives of  $\mathbf{F}$  exist and are continuous.” The next to the last line of the problem should read “... indeed, equivalent under the hypothesis that all partial derivatives exist and are continuous, assume ...”

## Excursion N.1 The Metric Space $C(K)$

- Page 410—in Lemma N.1.1, the definition of the metric should read

$$d(f, g) = \max_{x \in K} |f(x) - g(x)|.$$

## Excursion N.3 The Stone-Weierstrass Theorem

- Page 418—Problem 2. The third line of the problem reads, “there exists  $\delta > 0$  such that if  $t \in B_y(\delta)$ , then ...” It should, instead, read “there exists  $\delta > 0$  such that if  $t \in B_\delta(y)$ , then ...”
- Page 419—in step 2 of problem 4(b). The text should read: “Let  $x \in [0, 1]$ . Notice that if  $f(x) = x$ , then  $f(x) = f(x) - (f(x))^2 + x^2$ . In fact,  $f(x) = x$  is the unique non-negative fixed point for the function  $F : C[0, 1] \rightarrow C[0, 1]$  given by  $F(f) = f - f^2 + q$  where  $q$  is the quadratic function  $q(x) = x^2$  on  $[0, 1]$ .”
- Page 420—Steps 3 and 4 should be reversed.

## Excursion O.2 Picard Iteration

- Page 426—In problem 1,  $U$  must be convex. The problem should read, “Let  $U \subseteq \mathbb{R}^2$  be convex, and let  $f : U \rightarrow \mathbb{R} \dots$ ”
- Page 426—In problem 2. The displayed equation should match the corresponding equation on the previous page:

$$F(y)(t) = x_0 + \int_{t_0}^t f(u, y(u)) du.$$

## Excursion O.3 Systems of Equations

- Page 430—In problem 5. The constant  $\alpha$  mentioned in the result should be  $\alpha = \min \left\{ r, \frac{m}{nM} \right\}$ .

### Less important errors (more in the way of typos.)

- Page 45—The commas should be uniformly applied.
- Page 68—There is an extra comma after  $a, b, c$ .
- Page 80—In problem 11, the  $\bar{X}$  should just be  $X$ .
- Page 108—The word “approaches” in Theorem 4.2.3 should, instead, be “approaches.”

- Page 112—There should be a period at the end of the displayed equation on the very last line.
- Page 121—Exercise 5.1.3 currently reads “Let  $f$  and  $g$  from  $\mathbb{R}$  to  $\mathbb{R}$  such that ... It should, instead, read “Let  $f$  and  $g$  from  $\mathbb{R}$  to  $\mathbb{R}$  be such that ...”
- Page 132—The square brackets around the Hint at the end of problem 3(b) should, instead, be parentheses.
- Page 142—In problem 11, third line: “susequences” should be “subsequences.”
- Page 160—In the last paragraph, end of the first line, there is a comma after the word countably many. This comma shouldn’t be there.
- Page 210—In the footnote at the bottom of the page, second line. There should be a close parenthesis after the reference [McL].
- Page 212—In Theorem 11.2.3, in the second line we see “...that is a Riemann integrable on ...”. It should read, instead, “...that is Riemann integrable on ...”
- Page 244—The last line of the page needs a space between “functions” and “on.”
- Page 245—In problem 12, third line. The sentence at the end of the line that begins “The for all ...” should instead begin with “Then for all ...”
- Page 252—The first line of Corollary 12.4.5 ends in the word “is” and should, instead, end in the word “are.”
- Page 277—Theorem 13.4.8: The function referred to is a vector-valued function. Thus in lines 2 and 3,  $f$  should instead be  $\mathbf{f}$ .
- Page 282—Problem 3. The function  $f$  mentioned in the first line should, instead be  $\mathbf{f}$ , as it is a vector-valued function.
- Page 330—In problem 1, there should be a comma between “*non-convergent*” and “bounded.”
- Page 305—In Theorem C.1.2, the first line should read “Let  $a$  be a positive real number. Let  $r$  and  $s$  ...”
- Page 361—In problem 2, “Reimann” should, instead, be “Riemann.”