

Introduction

The Mathematical Association of America (MAA), through its Committee on the Undergraduate Program in Mathematics (CUPM), presents this *2015 CUPM Curriculum Guide to Majors in the Mathematical Sciences*. CUPM interprets its task broadly, both in affirming the breadth of possible offerings to undergraduate students majoring in mathematics and in understanding “mathematics” broadly to encompass what we call the mathematical sciences.

The roots of CUPM stretch back to the establishment of the Association in 1915. For many years CUPM has led MAA’s effort to guide the undergraduate curriculum in mathematics. In the October 2013 issue of the *American Mathematical Monthly*, Alan Tucker traces the history and development of the mathematics major in the US; this article is included in a rich electronic archive of CUPM’s work (www.maa.org/cupm). While the history of CUPM informs this *Guide*, we have tried to add information useful to today’s mathematics departments as they continue to shape vital and effective undergraduate curricula.

Curricular change occurs slowly. [Reshaping College Mathematics \(MAA Notes #13, 1989\)](#), [the 1991 CUPM report, *The Undergraduate Major in the Mathematical Sciences*](#), and [Confronting the Core Curriculum \(1998 MAA Notes # 45\)](#) all contain advice to the mathematical community with many of the ideas presented here. We urge faculty to read the [2001 CUPM Discussion Papers about Mathematics and the Mathematical Sciences in 2010: What Should Students Know?](#) to gauge how well their programs measure up to those expectations. From reports at focus groups and from data collected by CBMS surveys, we have found that departments have expanded their scope to more of the mathematical sciences, incorporating more statistics and probability, computing, discrete methods and operations research. Indeed, we see that many mathematical sciences programs are becoming more applied and more interdisciplinary. At the same time, publications of the National Academies of Science, have served to inform the mathematical community and the broader public about the ubiquity of mathematics.

The previous *CUPM Curriculum Guide* was published in 2004; it was followed by several related publications. Among them we call particular attention to the work of the CUPM Subcommittee on Curriculum Renewal Across the First Two Years (CRAFTY) on the [Curriculum Foundations Project](#), which emphasized the use of mathematics in other disciplines, and informs this *Guide*.

Recommendations in this *Guide* reflect CUPM’s reaffirmation of the principles in the 2004 *Guide*. Those principles, approved by MAA’s Board of Governors in 2003 and reapproved in August 2014 can be found [online](#). The 2004 *Guide* addressed the full range of mathematics offerings, including general education, service, and major courses. This *Guide* does not systematically address these non-major courses. This *Guide* focuses specifically on the design

of mathematics majors, addressing the curricular demands of the wide—and widening—variety of mathematics programs now found across the nation. That diversity often leads to minors, concentrations, double majors, and interdisciplinary majors, as well as full majors in new and developing mathematically rich fields. The purpose of this *Guide* is to help departments adapt their undergraduate curricula to this changing landscape while maintaining the essential components of the traditional mathematics major.

Many forces can be expected to drive curricular change. The adoption of the Common Core State Standards by many U.S. states, for instance, will significantly change the preparation of incoming college students and will necessitate changes in the preparation of pre-service teachers. Two-year colleges have seen rising enrollments; many of their graduates transfer to four-year institutions. The publications of the American Mathematics Association of Two-Year Colleges (AMATYC) and the work of MAA's own Committee on Two-Year Colleges and its Committee on Curriculum Renewal Across the First Two Years (CRAFTY) have provided guidance over the years on the mathematics of the first two years and on developmental courses in the subject. Recent publications, such as that of the [INGenIOuS Project](#), urge mathematics departments to prepare students for the modern workforce. [Transforming Post-Secondary Education in Mathematics](#) (APSE), a project sponsored jointly by the Carnegie Corporation of New York and the Alfred P. Sloan Foundation, urges constructive change in mathematics education at community colleges, 4-year colleges, and research universities, especially in the first two years.

CUPM urges departments to think creatively about their introductory courses. Departments might consider revisions to their developmental programs that could attract underprepared students to take more mathematics, perhaps even leading to a minor in the subject. Such programs as Quantway, Statway, and the newly developed Mathway are examples of such mainstreaming and just-in-time courses for those students. Students who have Advanced Placement credit in mathematics should be offered interesting courses that explore the nature of modern mathematics and its applications. Students are attracted to mathematics for its beauty, its utility, and its intellectual merits. National efforts should be made to develop materials for challenging and interesting courses that include bridges and alternate pathways to the major.

New uses for mathematics in many disciplines have emerged in the decade since the 2004 *Guide*. The reports [The Mathematical Sciences in 2025](#) and [Fueling Innovation and Discovery: The Mathematical Sciences in the 21st Century](#), both publications of the Board of Mathematical Sciences and Their Applications of the National Academies, explore new areas of the physical, biological, and geophysical sciences; and of engineering, computing, and social sciences in which mathematics will play an essential part. The 2012 PCAST report, [Engage to Excel](#), described strategies for producing one million additional degrees in STEM fields. This *Guide* is a partial response to the call for increasing the number of STEM degrees. The *2015 Curriculum Guide* focuses on undergraduate programs that are mathematically intensive and offers recommendations on the mathematics major and on mathematical sciences programs that tend to be interdisciplinary.

As work progressed on this *Guide*, the education committees of both the American Statistical Association (ASA) and the Society for Industrial and Applied Mathematics (SIAM) were preparing their own recommendations for undergraduate degrees in statistics and applied mathematics, respectively. CUPM has benefited from their work and hopes for continued consultation and cooperation among our societies.

Preparation for this *Guide*. Work on this *Guide* began around 2009, when CUPM began sponsoring focus group discussions to assess the community’s general need for a new curriculum guide and, in particular, to gauge interest in addressing the variety of possible undergraduate paths to a major in mathematics. CUPM also began, through surveys and focus groups, to identify essential components of interdisciplinary majors that might be described as majors in a mathematical science. Broad agreement seems to exist on “core” elements of both a traditional mathematics program and a broader mathematical sciences major. We attempt to capture these elements in the Overview document that follows.

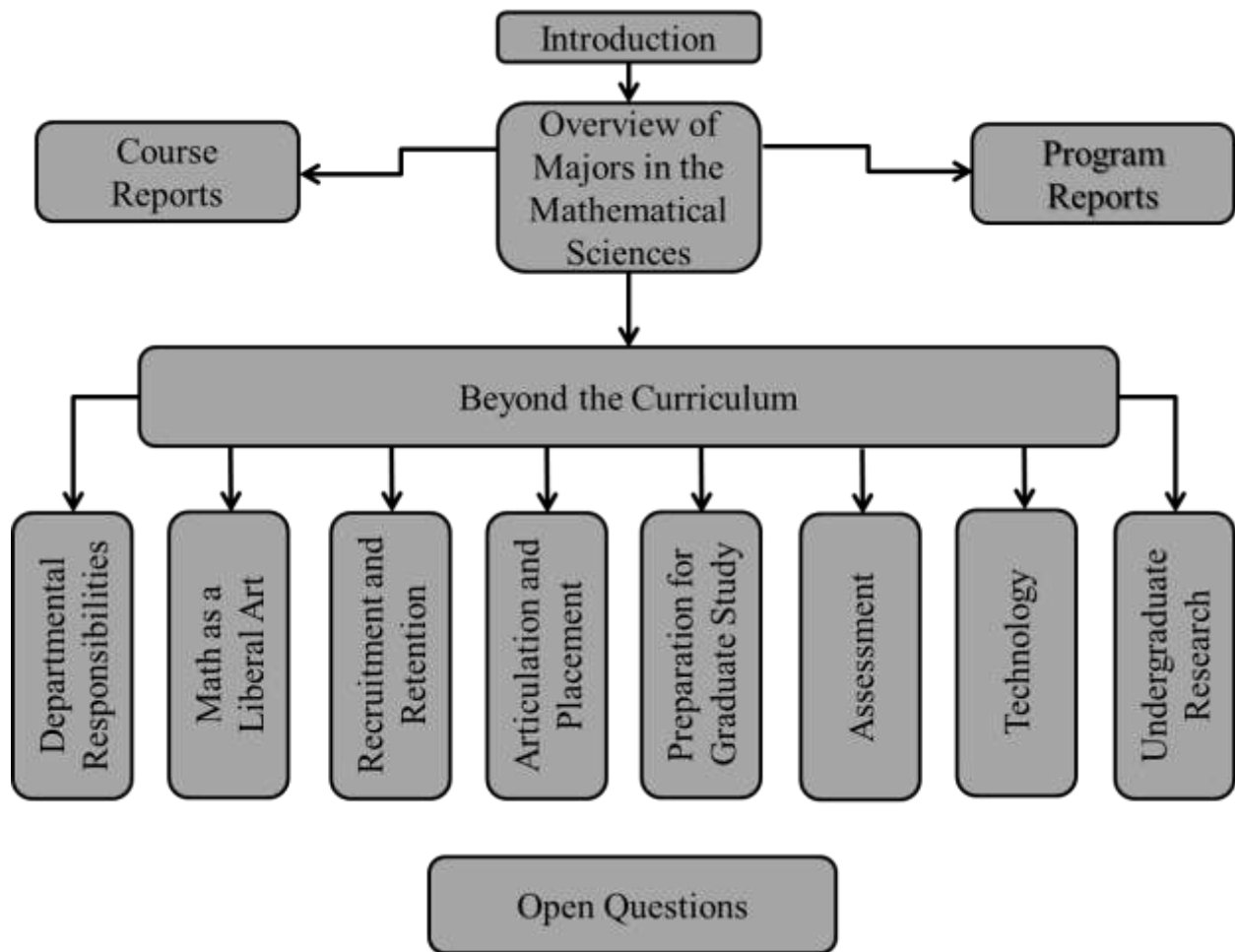
Although the *Guide* is a project of CUPM, a Steering Committee (members are listed at the end of this introduction) was appointed to refine the conceptual framework of the project and to manage its development. The Steering Committee helped in selecting the many small study groups contributing to this *Guide* and in improving their reports. It was agreed to produce both a relatively brief printed *2015 Curriculum Guide to Majors in the Mathematical Sciences* and a more extended online set of companion resources. The title reflects CUPM’s intention to consider not only the “traditional” mathematics major but also programs that serve students who pursue (through majors, minors, concentrations, etc.) degrees across the range of mathematical sciences.

It was also agreed that this *Guide* would focus mainly on the curriculum itself, without aiming to discuss in detail the full set of related issues, including pedagogy, access, technology, articulation, placement, and diversity. Nevertheless, we acknowledge the critical importance of addressing these issues in delivering a curriculum effectively; for this reason we have included several brief reports on these matters in the section called Beyond the Curriculum. Where MAA committees have done recent work in crucial but non-curricular areas that affect teaching and learning, committee summaries are included as reports in that section. Where possible, we have cited models of effective programs, new courses, or best practices we recommend for investigation by departments aiming to improve in these areas.

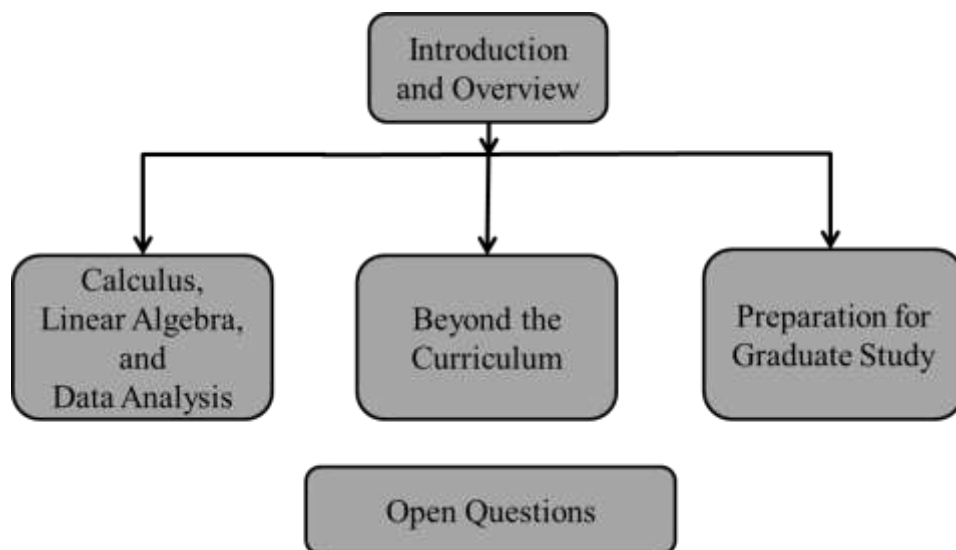
While this *Guide* pays only secondary attention to pedagogy, we fully acknowledge that the subject deserves its own serious attention and discussion in the community. MAA’s Committee on the Teaching of Undergraduate Mathematics has launched its own study and will publish its own report on the growing body of knowledge on the pedagogy of collegiate mathematics education. Pedagogy and the curriculum are so closely related that the Cognitive Goals in this *Guide* were developed to reflect CUPM’s firm belief that the major in any mathematical science

should promote very special habits of mind. The intentional cultivation of these mathematical habits of mind takes precedence over a strict syllabus in all courses. The close connection between pedagogy and the curriculum is also reflected in the parallel cognitive and content recommendations in the Overview.

How to use this Guide. This *Guide* will be delivered in two forms: an abbreviated print version and a longer and dynamic online version. There are six parts to this report: Introduction, Overview of the Major, Course Area Group Reports, Program Area Group Reports, Beyond the Curriculum, and Conclusion. The diagram below describes their organization. The entire document and references will be available on maa.org/cupm.



The printed version includes the Introduction; Overview; the Calculus, Linear Algebra, and Data Analysis Course Area Group Reports; Preparation for Graduate Education; Beyond the Curriculum; and the Conclusion and Open Questions.



Introduction and Overview. Everyone should read this Introduction and the Overview. The Overview contains the cognitive and content recommendations for all mathematical sciences major programs. CUPM regards these recommendations, approved by the MAA Board of Governors in August 2014, as the core of a mathematics major of any type.

The Overview was prepared after sustained consultation with MAA members and with the mathematical community at large. Several focus groups met at MathFest 2013 and offered constructive suggestions on a draft. At JMM 2014, CUPM, the Board of Governors and undergraduate program chairs from across the country contributed their views on the revised Overview. Member organizations of CBMS were also invited to comment on the Overview and on their choice of any documents in this *Guide*. The final form of these recommendations reflects the wisdom and experience of many contributors.

The Overview aims to help frame discussion for the design of any degree in the mathematical sciences. It is based on the premise that every department should develop and articulate a set of goals, and should intentionally craft a program that helps its students meet those goals. We recommend that departments discuss both cognitive and content goals and how they relate to each other and to the department's programs. We emphasize that there are many ways to craft a major: we see one type of program as treating mathematics as a liberal art, another concentrating on applied mathematics in general, another preparing students for professional work (such as teacher education or actuarial science), or preparing students for graduate school. We believe that our recommended cognitive and content goals form the framework of every one of these programs.

Every department and every institution has a unique mission. Curricular planning in any department should therefore be informed by recommendations from the mathematical community as well as by local knowledge of students, faculty, resources, administration, and, where they apply, governmental constraints and support. We emphasize that in order to attain its

cognitive goals, a department must design not only courses but also specific exercises and activities that are constructed explicitly to advance students to the next cognitive level.

Every department should have a regular schedule of program review and self-study. The assessment of a department's progress should be regular and pre-planned, with data collected and analyzed systematically. Carefully facilitated departmental retreats for small departments (or for subgroups of large departments) work well for designing curricula to meet established goals. MAA's Committee on Departmental Review has prepared [materials](#) to assist departments with self-studies and periodic reviews.

CUPM has attempted to integrate recommendations for the traditional mathematics major program that often strove to prepare all students for graduate study with recommendations for a major that encourages students who love mathematics to apply their abilities to those fields in which mathematics plays an increasingly important role. To that end we have reports on many program areas and the many courses that support them.

Course Area Study Groups (CASGs). Focus groups held as early as 2009 revealed that the names of common courses such as Real Analysis and Linear Algebra mean different things to different people. Discussions sometimes foundered over terms that, although "locally well defined," were understood differently among participants. This persistent miscommunication led to the formation of CASGs.

CASGs were asked to present several reasonable versions of courses, each with its natural student audience and, where possible, a few references. Several standing MAA committees, such as the Committee on Mathematics Across the Disciplines (MAD) and the ASA-MAA Joint Committee on Statistics, made substantial contributions to course area reports. We urged CASGs to consider cognitive goals along with content, and we welcomed diversity of viewpoint and discouraged votes to "prefer" one type of course over another.

Each CASG had three to six members, each with expertise in the area. Some CASG members are not MAA members; some are not mathematicians, though each group member has considerable experience with the subject matter. To get a broad view of courses we invited professors, users, and researchers in these areas. Some members are textbook authors, familiar with the range and level of courses offered in a given area in a wide variety of schools.

Program Area Study Groups (PASGs). PASGs were invited to describe programs in a variety of mathematical sciences as well as in applied areas, usually within undergraduate departments. Several directors of graduate programs were asked for their perspectives on what makes good undergraduate preparation for graduate study in the field. A typical PASG included four to seven members, representing different types of schools and departments. Many members had initiated or run a major track, a full major, a minor, or a small concentration in the given area. Some non-mathematicians also served on PASGs, representing their disciplines. In many cases PASGs have offered suggestions for a full-fledged joint major involving mathematics and

another discipline. In those areas we have included recommendations for adaptations for those departments with limited resources or fewer faculty and students.

One of the most important responsibilities of many mathematics departments is the preparation of teachers. This *Guide* features the recommendations of MAA's Committee on the Education of Teachers (COMET) for a professional program for the preparation of pre-service middle school and secondary school teachers of mathematics. Implementation of the program that is more of a professional track than the traditional liberal arts track is carefully discussed. The report marks the most significant changes in COMET's recommendations for the mathematical education of pre-service middle and high school teachers in 25 years.

PASG reports reflect the work of many entities. MAA SIGMAAs (special interest groups) were invited to contribute to reports in their areas. MAA's Committee on the Mathematical Education of Teachers (COMET) and the ASA-MAA Joint Committee on Statistics were largely responsible for the reports on teacher education and statistics, respectively. We also relied heavily on members of the Society of Actuaries, the ASA, the Association for Computing Machinery, the American Chemical Society, the Society for Mathematical Biology, the American Mathematical Society, the National Council of Teachers of Mathematics, and SIAM. We thank our many colleagues in other STEM disciplines for their assistance.

Beyond the Curriculum. Mathematics is more than curriculum. It is also a human activity. This *Guide* is principally about curricular and cognitive elements in the teaching and learning of mathematics, but we acknowledge that many social factors also play important roles in mathematics classrooms. Whenever we design curricula or single courses, we should consider students' backgrounds and what they bring into our courses or programs—as well as what we would like them to learn. Reports on building community and culture as well as ideas about recruitment and retention of majors were written by mathematicians with well-known, successful programs, especially for underrepresented groups.

Most of the other “Beyond the Curriculum” reports in this *Guide* are summaries from various MAA committees. The MAA's Committees on Assessment, Articulation and Placement, Research by Undergraduates, and Teaching of Undergraduate Mathematics all contributed to this report. A special focus group on Technology and the Mathematics Curriculum held at Mathfest 2014 led to the formation of a working group on that subject. While national professional norms and accreditation standards, federal guidelines, state laws, institutional requirements, and student interests and abilities influence the curricular process, the *department* is the unit that serves as the primary leader in effecting actual change in the curriculum and beyond. The Departmental Responsibilities section points to the major elements necessary for a department's systematic approach to carrying out its mission.

Online resources. Reports in the *Guide* point to many online documents and resources. We think of the report and its recommendations as evolving with time. CUPM expects these

resources to be helpful now, but the committee plans to have them regularly updated by CASG- and PASG-type interest groups, and by the community at large. CUPM and the Steering Committee have suggested that MAA form online communities in both course and program areas, building toward rich, current resources of ideas, course modules, software, and other curricular materials.

CUPM in the future. CUPM is fully aware that in a changing mathematical world any static curriculum guide is necessarily incomplete and soon out of date. The current committee strongly recommends that CUPM, its subcommittees, or appropriate *ad hoc* task forces be charged with maintaining a dynamic document based on this *Guide*. Both the Steering Committee and the current members of CUPM suggest that CUPM be charged to update this document every year, adding or revising sections on a rolling basis. Materials from and examples of effective programs that have implemented recommendations in CASG and PASG reports will be especially useful to CUPM. CUPM or its designates should solicit, review, update, and share such contributions with the MAA membership. Compiling such “models that work” is an important extension of this *Guide*.

A word about Calculus. CUPM cannot conceive of a mathematics major without some part of its curriculum devoted to ideas of calculus. That said, CUPM is troubled both by the level of students’ pre-calculus preparation and by the permanent exodus from mathematics of many good students who receive college credit for high school or AP calculus or are discouraged by their experience in college calculus classes. We know anecdotally that many mathematics majors begin college work with either Calculus II or Multivariable Calculus, but our present knowledge is scanty as regards what calculus our major students encounter, and when.

CUPM began this project at the same time that David Bressoud and colleagues were collecting data for the MAA research project on Calculus I, *Characteristics of Successful Programs in College Calculus (CSPCC)*. The CASG report on Calculus draws on some information from the CSPCC study, and a separate report on the CSPCC study appears in the Calculus CASG report. Together, these reports present some promising recent initiatives in and live questions about calculus within the major curriculum. Over time we hope for additional research, careful studies of Calculus II and Calculus III that both mirror and follow up the *CSPCC*, more examples of innovative courses, and more “proofs” of curricular success.

Thanks. When this project was conceived, Carol Schumacher (Kenyon College) chaired CUPM. She deserves much of the credit for the initiation and development of this project, which she now co-chairs with Martha Siegel, the current chair of CUPM. Michael Pearson and Linda Braddy of the MAA have given their unwavering support. Thanks also go to the members of the Steering Committee: Betsy Yanik (Emporia State University), David Bressoud (Macalester College), Jenna Carpenter (Louisiana Tech University), Michael Starbird (University of Texas, Austin), Alan Tucker (Stony Brook University), and Harriet Pollatsek (Mount Holyoke College). In addition, Beth Burroughs (University of Montana) and Joseph Malkevitch (York

University of the City of New York), chairs of COMET and MAD, respectively, gave generously of their time and their wisdom. We were also fortunate to have Paul Zorn (St. Olaf College) and Harriet Pollatsek (Mount Holyoke College) as editors.

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To everyone named above and to the many volunteer professionals who contributed work and wisdom to course, program, and other reports, we offer our sincere thanks. Members of the community spent many hours researching and writing their group reports. Member organizations of the Conference Board on Mathematical Sciences (CBMS) made significant contributions in reviewing the *Guide*. Our course and program groups were gracious in responding to editorial comments and made valuable and timely revisions.

Comments, suggestions, and corrections should be directed to

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