Solutions to Holdener Review packet

#1 a) $f_x(S) < f_y(Q) < 0 < f_x(P) < f_y(R)$

b) $f_{xx}(P) > 0$ because the slice of $z = f(x,y)$ parallel to the x-axis at P is concave up.

#2. $$f_x(x,y) = \frac{\sqrt{x^2+y^2}\cdot 2y^2 - 2xy^2 \cdot \frac{1}{2}\cdot\frac{1}{\sqrt{x^2+y^2}}(2x)}{x^2+y^2}$$

#3. $f(x,y) = x^3 + 6xy - 2y^2 + 1$, $f_{\vec{u}}(1,1) = ?$

$\nabla f(x,y) = \langle 3x^2+6y, 6x-4y \rangle$

$\nabla f(1,1) = \langle 9, 2 \rangle \quad \hat{u} = \left\langle \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right\rangle$

$$f_{\vec{u}}(1,1) = \langle 9, 2\rangle \cdot \left\langle \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right\rangle = \frac{18}{\sqrt{5}} + \frac{2}{\sqrt{5}} = \frac{20}{\sqrt{5}} = \boxed{4\sqrt{5}}$$

#4. To say $f_{\vec{u}} = \nabla f$ is complete nonsense because $f_{\vec{u}}$ is a number (the slope of $z=f(x,y)$ at P in the direction of $\vec{u}$) while ∇f is a vector (the direction of greatest initial increase of f at P). Bubba should have stated that $f_{\vec{u}} = \|\nabla f\|$. (Indeed, in the situation given, $\vec{u}$ is perpendicular to the contour at P.)

#5 The differential df is defined by $df = f_x\,dx + f_y\,dy$. df estimates $\Delta f = f(x+dx, y+dy) - f(x,y)$ for small dx & dy.

#6. Informally, we say that f is differentiable at a point (a,b) if f is locally planar at (a,b). Formally, f is differentiable if

$$\lim_{\substack{h\to 0\\ k\to 0}} \frac{E(a+h,\, b+k)}{\sqrt{h^2+k^2}} = 0$$ when $f(x,y) = L(x,y) + E(x,y)$ & $L(x,y)$ is the tangent plane at (a,b).

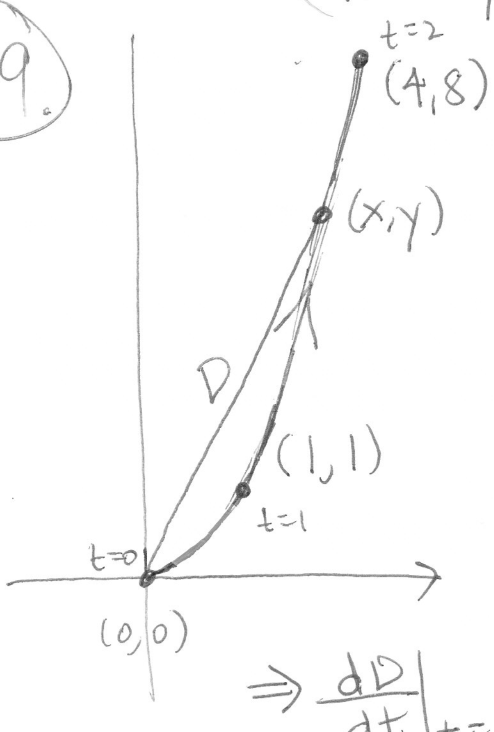
#7. False. The partials existing only says something about the slope in <u>two</u> directions. Differentiability means the function is locally planar, so the slopes in various directions at a point must reflect the shape of a (tangent) plane at that point.

#8. $$\left(\frac{\partial z}{\partial s}\right)\left(\frac{\partial z}{\partial t}\right) = \left(\frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial s}\right)\left(\frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial t}\right)$$

$$= \left(f_x\cdot 1 + f_y\cdot 1\right)\left(f_x\cdot 1 + f_y\cdot(-1)\right)$$

$$= \left(f_x + f_y\right)\left(f_x - f_y\right) = f_x^{\,2} - f_y^{\,2}.$$

#9.



$$D(x,y) = \sqrt{x^2+y^2}$$

You're asked to compute $\left.\frac{dD}{dt}\right|_{t=1}$ when $x(t) = t^2$ and $y(t) = t^3$.

$$\frac{dD}{dt} = \frac{\partial D}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial D}{\partial y}\cdot\frac{dy}{dt}$$

$$= \frac{1}{2}\cdot\frac{2x}{\sqrt{x^2+y^2}}\cdot 2t + \frac{1}{2}\,\frac{2y}{\sqrt{x^2+y^2}}\cdot 3t^2$$

$$= \frac{1}{2}\,\frac{2t^2}{\sqrt{t^4+t^6}}\cdot 2t + \frac{1}{2}\cdot\frac{2t^3}{\sqrt{t^4+t^6}}\cdot 3t^2$$

$$\Rightarrow \left.\frac{dD}{dt}\right|_{t=1} = \frac{2}{\sqrt{2}} + \frac{3}{\sqrt{2}} = \boxed{5/\sqrt{2}}$$

#10 $\nabla f(w,T) = \langle 0.16(-35.75)w^{-0.84} + 0.16(0.4275)Tw^{-0.84},\ 0.6215 + 0.4275w^{0.16} \rangle$

a) $\nabla f(5,30) = \langle 0.16(-35.75)5^{-0.84} + 0.16(0.4275)(30)(5)^{-0.84},\ 0.6215 + (0.4275)5^{0.16} \rangle$

$\approx \langle -0.949, 1.175 \rangle$

b) The wind chill increases at the greatest rate in the direction of $\langle -.949, 1.175 \rangle$ when $w = 5$ mph and $T = 30\,°F$.

c) $|f_T(5,30)| > |f_w(5,30)|$ so a decrease in the temperature from $30°F$ would have a greater effect on the windchill than an increase in the wind speed.

#11 $\frac{\partial Q}{\partial K} = \alpha b K^{\alpha-1}L^{1-\alpha}$ & $\frac{\partial Q}{\partial L} = (1-\alpha)bK^{\alpha}L^{-\alpha}$

So $K\frac{\partial Q}{\partial K} + L\frac{\partial Q}{\partial L} = K(\alpha b K^{\alpha-1}L^{1-\alpha}) + L((1-\alpha)bK^{\alpha}L^{-\alpha})$

$= \alpha b K^{\alpha}L^{1-\alpha} + (1-\alpha)bK^{\alpha}L^{1-\alpha}$

$= (\alpha b + (1-\alpha)b)K^{\alpha}L^{1-\alpha} = bK^{\alpha}L^{1-\alpha} = Q.$

#12 a) Use the differential: $dT = T_x\,dx + T_y\,dy$

$\Delta T \approx dT = 4\cdot(-0.02) + 3(0.03) = -0.08 + 0.09$
$= 0.01$

So $T(1.98, 3.03) = T(2,3) + \Delta T$
$\approx T(2,3) + dT$
$= 65 + 0.01 = \underline{65.01}.$

#13 Direction vector: $\langle 5, -8 \rangle$

a) Line segment: $\langle 2,3 \rangle + t\langle 5,-8 \rangle$

$$\Rightarrow \begin{aligned} x(t) &= 2+5t \\ y(t) &= 3-8t \end{aligned} \quad \Rightarrow (x(0), y(0)) = (2,3) \;\&\; (x(1), y(1)) = (7,-5).$$

b) You need to slow the motion down by a factor of 4 because it takes 4 units of time to traverse the same line segment.

So replace t w/ $t/4$...

You get $x(t) = 2 + \frac{5}{4}t$ & $y(t) = 3 - 2t$

Then $(x(0), y(0)) = (2,3)$ & $(x(4), y(4)) = (7,-5)$.

#14 a) Both $f_x(0,0)$ & $f_y(0,0)$ exist.

$$f_x(0,0) = \lim_{h\to 0} \frac{f(0+h,0) - f(0,0)}{h} = \lim_{h\to 0} \frac{f(h,0)-f(0,0)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{2h\cdot 0}{(h^2+0^2)^2} - 0}{h} = \lim_{h\to 0} \frac{0}{h} = 0$$

Similarly, $f_y(0,0) = 0$. Note: Because f is piecewise defined at $(0,0)$, you must use the formal definition of the partial derivative to compute $f_x(0,0)$ and $f_y(0,0)$. If you were asked to compute f_x & f_y at any point other than $(0,0)$, then you could have computed them by applying the quotient rule to $2xy/(x^2+y^2)^2$ (and f_x, f_y would exist).

b) No $\lim_{\substack{(x,y)\to(0,0) \\ \text{along } y=x}} f(x,y) = \lim_{x\to 0} \frac{2x^2}{(x^2+x^2)^2} = \lim_{x\to 0} \frac{1}{2x^2}$ DNE If limit doesn't exist f can't be continuous.

c) No. f differentiable implies f is continuous.
f not continuous implies f is not differentiable.