Math 336 - Probability

Exam #1 Solutions

1. Let X = # of passengers who show up for their flight.

$X \sim B(n=110, p=.9)$

a.) $P(X > 100) = 1 - P(X \leq 100) \underset{\text{MTB}}{=} 1 - .6710 = .329$

OR

$P(X > 100) \approx 1 - P\left(Z \leq \frac{100+.5 - 110(.9)}{\sqrt{110(.9)(.1)}}\right)$

$\boxed{\begin{array}{l} 110(.9) = 99 \geq 10 \checkmark \\ 110(.1) = 11 \geq 10 \checkmark \end{array}}$

$= 1 - P(Z \leq .4767)$

$= 1 - .6832 = .3168$

Without C.C. $1 - .6247 = .3753$

b.) If people travel in groups and the probability of showing up stays the same, $p = .9$, then this is essentially the same as reducing the sample size. Since smaller samples have more variation, the tails will have more probability. Thus, the probability in part (a), which is a right tail probability, will increase.

2. a.) X = call on which the second sale is made.

The possible values for X are 2, 3, 4, ... OR integers larger than one.

b.) $P(X=5) = P(\text{1 sale in first 4 calls} \cap \text{2}^{\text{nd}} \text{ sale on 5}^{\text{th}} \text{ call})$

$\underset{\text{indep}}{=} \binom{4}{1}(.2)^1(.8)^3 \times .2$

$= 4(.2)^2(.8)^3 = .0819$

c.) $P(X=k) = P(\text{1 sale in first } k-1 \text{ calls} \cap \text{2}^{\text{nd}} \text{ sale on } k^{\text{th}} \text{ call})$

$= \binom{k-1}{1}(.2)^1(.8)^{k-2} \times .2$

$= \binom{k-1}{1}(.2)^2(.8)^{k-2}, \; k = 2, 3, \ldots$

#1 b

$Y \sim Bin(2, .9)$

$P(\text{overbooked}) = P(Y > 1) = 1 - P(Y \le 1)$

$= 1 - \left[\binom{2}{0}(.9)^0(.1)^2 + \binom{2}{1}(.9)^1(.1)^1\right]$

$= 1 - [.1^2 + 2(.9)(.1)]$

$= 1 - .19 = .81$

$Y \sim B(10, .9)$ 10 groups of 11

$P(\text{overbooked}) = P(Y > 9) = 1 - P(Y \le 9)$

$= 1 - .6513 = .3487$

$Y \sim B(55, .9)$

$P(\text{overbooked}) = P(Y > 50) = 1 - P(Y \le 50)$

$= 1 - .6549 = .3451$

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3. (a.) 50 T/F + 80 MC = 130 possible questions / We must select 25 for an exam

$$\binom{130}{25} = 3.8553 \times 10^{26} \text{ possible exams.}$$

(b.) $$\binom{50}{13}\binom{80}{12} = (3.5486 \times 10^{11}) \times (6.0247 \times 10^{13})$$
$$= 2.1379 \times 10^{25}$$

(c) $$\frac{\binom{50}{0}\binom{80}{25}}{\binom{130}{25}} = \frac{3.6341 \times 10^{20}}{3.8553 \times 10^{26}} = 9.4262 \times 10^{-7}$$
(very small)

4. (a) $$\bigcup_{n=1}^{\infty} \left[3-\tfrac{1}{n}, 3+\tfrac{1}{n}\right] = [2,4] \cup [2.5, 3.5] \cup [2\tfrac{2}{3}, 3\tfrac{1}{3}] \cup \cdots$$
$$= [2,4]$$

(b) $$\bigcap_{n=1}^{\infty} \left(3-\tfrac{1}{n}, 3+\tfrac{1}{n}\right) = (2,4) \cap (2.5, 3.5) \cap (2\tfrac{2}{3}, 3\tfrac{1}{3}) \cap \cdots$$
$$(2\tfrac{n-1}{n}, 3\tfrac{1}{n}) \cap \cdots$$
$$= \{3\}$$

5. Let $L = \{$the person selected has lung disease$\}$
$S = \{$ the person selected is a smoker $\}$

Given: $P(L) = .07$, $P(S|L) = .9$, and $P(S^c|L^c) = .747$

Find $$P(L|S) = \frac{P(S|L)\,P(L)}{P(S|L)\,P(L) + P(S|L^c)\,P(L^c)}$$

$$= \frac{.9(.07)}{.9(.07) + (1-.747)(1-.07)}$$

$$= \frac{.063}{.063 + .2353} = .2112$$

Approx 21% of smokers have lung cancer

Math 336 - Probability

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Extra Credit: Prove that $E \cup (F \cap G) = (E \cup F) \cap (E \cup G)$

($\Rightarrow$) Suppose that $\omega \in E \cup (F \cap G)$. This implies that $\omega \in E$ or $\omega \in (F \cap G)$. If $\omega \in E$, then $\omega \in E \cup F$ and $\omega \in E \cup G$. If $\omega \notin E$, then $\omega \in (F \cap G)$. Thus, $\omega \in F$ so $\omega \in E \cup F$ <u>and</u> $\omega \in G$ so $\omega \in E \cup G$. Either way, $\omega \in (E \cup F) \cap (E \cup G)$.

($\Leftarrow$) Suppose $\omega \in (E \cup F) \cap (E \cup G)$. Then $\omega \in E \cup F$ and $\omega \in E \cup G$.
If $\omega \in E$, then $\omega \in E \cup (F \cap G)$.
If $\omega \notin E$, then $\omega \in F$ and $\omega \in G$ which implies $\omega \in E \cup (F \cap G)$.
Thus, $\omega \in E \cup (F \cap G)$ in both possible cases.

We have shown $E \cup (F \cap G) \subseteq (E \cup F) \cap (E \cup G)$ and $(E \cup F) \cap (E \cup G) \subseteq E \cup (F \cap G)$.
Therefore, the two sets are equal.