132 Possible Points

Math 336 - Exam 1 Solutions (Fall 2002)

1. Use multinomial coefficient, with 3 groups $N=18,\ n_1=5,\ n_2=6,\ n_3=7$

$$\frac{18!}{5!6!7!} = 14,702,688$$

2. Given: $P(ND|A) = .08$ $\quad P(AD|A) = .92$

$P(AD|N) = .05$ $\quad P(ND|N) = .95$

$P(A) = .02 \Rightarrow P(N) = 1 - P(A) = .98$

(a) $P(AD) = P(AD|N)P(N) + P(AD|A)P(A)$

$= .05(.98) + .92(.02) = .049 + .0184 = \boxed{.0674}$

(b) $P(N|AD) = \dfrac{P(AD|N)P(N)}{P(AD)} = \dfrac{.05(.98)}{.0674} = \dfrac{.049}{.0674} = \boxed{.7270}$ **

$P(A|AD) = \dfrac{P(AD|A)P(A)}{P(AD)} = \dfrac{.92(.02)}{.0674} = \boxed{.273}$ **

(c) $P(ND) = P(ND|A)P(A) + P(ND|N)P(N)$

$= .08(.02) + .95(.98) = .9326$

$P(N|ND) = \dfrac{P(ND|N)P(N)}{P(ND)} = \dfrac{.95(.98)}{.9326} = \boxed{.9983}$

$P(A|ND) = \dfrac{P(ND|A)P(A)}{P(ND)} = \dfrac{.08(.02)}{.9326} = \boxed{.0017}$

Dean (d) The probabilities in part (b) are troubling. Approximately 73% of those children classified as abused by a doctor are not being abused.

3. $P(\text{at least one match}) = 1 - P(\text{no matches})$

$= 1 - P(\text{not } i \text{ on } i^{th} \text{ trial},\ i=1,\dots,n)$

$= 1 - \left(\frac{n-1}{n}\right)^n = \boxed{1 - \left(1 - \frac{1}{n}\right)^n}$

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#4. Let X = # of winning tickets.

$X \simeq Bin(n; p = .1)$ Must check assumption or comment on approx.

a.) $P(X \geq 1) \simeq 1 - P(X=0)$ ← Exact probability can be calculated with the Hypergeometric Distribution.

$\simeq 1 - \binom{10}{0}(.1)^0(.9)^{10}$

$= 1 - .3487 = \boxed{.6513}$

b.) We want to find the smallest n such that

$P(X \geq 1) \geq .5$

Matt. Using the binomial approximation, we want to find n such that

$$1-(.9)^n \geq .5 \iff -(.9)^n \geq -.5$$
$$\iff (.9)^n \leq .5$$
$$\iff n \ln(.9) \leq \ln(.5)$$
$$\iff n \geq \frac{\ln(.5)}{\ln(.9)} = 6.5788$$

Purchase $\boxed{n=7}$ tickets!

#5. Let X = # of trials until the first success (same B-day as yours)

(a) $X \sim$ Geometric $(p = \frac{1}{365})$

$$P(X=x) = \begin{cases} \left(\frac{364}{365}\right)^{x-1}\left(\frac{1}{365}\right), & x = 1, 2, \ldots \quad (\text{or } x \geq 1) \\ 0, & \text{elsewhere} \end{cases}$$

(b) Use geometric series: $\sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}$

(i) $\left(\frac{364}{365}\right)^{x-1}\left(\frac{1}{365}\right) \geq 0 \quad \forall x \geq 1$ ✓

Lisa.

(ii) $$\sum_{x=1}^{\infty}\left(\frac{364}{365}\right)^{x-1}\left(\frac{1}{365}\right) \underset{\text{reindex the sum}}{=} \sum_{y=0}^{\infty}\left(\frac{364}{365}\right)^{y}\left(\frac{1}{365}\right)$$

$$\underset{\text{Geometric series}}{=} \frac{1/365}{1 - 364/365} = \frac{1/365}{1/365} = 1$$

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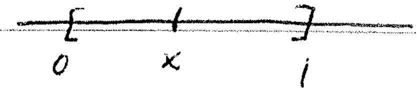
#6. Consider the interval $[0,1]$



$$P\{\underbrace{\max(1-x,x) \geq 2 * \min(1-x,x)}\}$$

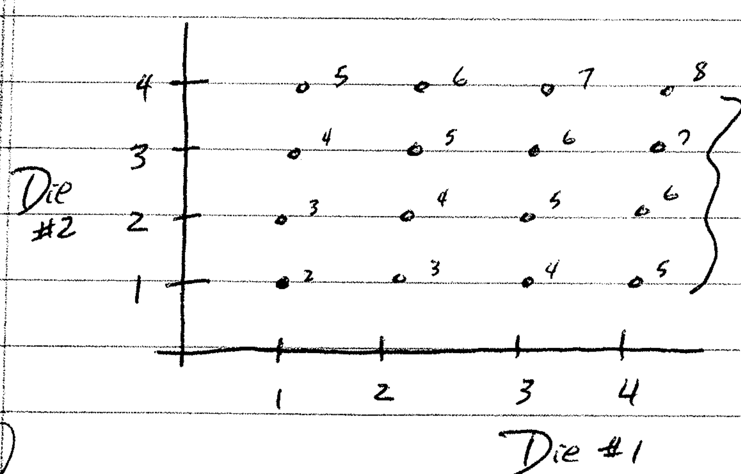
$\to x \geq 2(1-x) \iff x \geq 2-2x \iff 3x \geq 2 \iff \boxed{x \geq 2/3}$

$\to 1-x \geq 2x \iff 1 \geq 3x \iff \boxed{\frac{1}{3} \geq x}$

$$= P\left(x \in (0,\tfrac{1}{3}) \text{ } \underline{\underline{OR}} \text{ } x \in (\tfrac{2}{3},1)\right)$$

$$= \frac{1}{3} + \frac{1}{3} = \boxed{\frac{2}{3}}$$

#7.



16 possible outcomes are equally likely

(a)

S	2	3	4	5	6	7	8
prob	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

(b)

D	0	1	2	3
prob	$4/16$	$6/16$	$4/16$	$2/16$

(c)

L	1	2	3	4
prob	$1/16$	$3/16$	$5/16$	$7/16$

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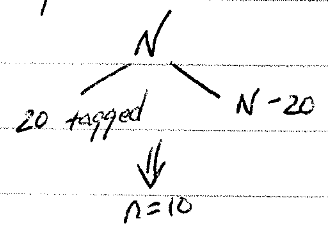
#8. Let T = # of tagged fish in the 2nd sample

a.) Hypergeometric Distribution



$$P(T=t) = \frac{\binom{20}{t}\binom{N-20}{10-t}}{\binom{N}{10}} \quad t=0,\dots,10$$

b.) Binomial Dist with $n=10$ and $p=\frac{20}{N}$

$$P(T=t) = \binom{10}{t}\left(\frac{20}{N}\right)^t\left(1-\frac{20}{N}\right)^{10-t} \quad t=0,\dots,10$$

#9 Josh.