Math 224 **Name:** ______________________

Final Exam Part II **15 December 2006**

INSTRUCTIONS: Complete the following problems without the aid of any person or any outside reference material. Use of calculators and computer algebra systems such as Maple is permitted.

1. (9 points) Determine if the set $H = \{(x, y, z) : x - 4y - z = 0\}$ is a subspace of $\Re^3$.

2. (9 points) Let $\vec{v}_1 = (1, -1, 1, 1)$, $\vec{v}_2 = (3, 2, 2, 3)$, $\vec{v}_3 = (1, 4, 0, 1)$, and $\vec{v}_4 = (-1, 3, 1, 2)$. Find a basis for the span of $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ and the dimension of the space.

3. (6 points) Determine whether the set $\left\{ \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\}$ is linearly independent in M_{22}.

4. (12 points) Let $T : \Re^3 \rightarrow \Re^3$ be given by $T(x, y, z) = (2x - y, 2x + y, x + z)$.

   (a) Is T a linear transformation? If so, what is $[T]$, the standard matrix representation of T?

   (b) Does T^{-1} exist? If so, what is $[T^{-1}]$?

5. (20 points) Let $A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 4 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 2 & 4 \end{bmatrix}$.

(a) Calculate the characteristic polynomial of A. You may use Maple or Mathematica or a calculator to compute a determinant, but please do not simply ask for the characteristic polynomial of A.

(b) Calculate the eigenvalues of A by factoring the characteristic polynomial of A.

(c) Calculate bases for the eigenspaces of A by row reduction of the appropriate matrices.

(d) What are the algebraic and geometric multiplicity of each eigenvalue of A?

6. (6 points) By what matrix would you multiply a vector $\vec{v}$ to change it from the standard basis to the basis $B = \{(1,0,1),(0,-1,2),(2,3,-5)\}$?

7. (8 points) Let W be a subspace of inner product space V. Show that $W^{\perp}$ is a subspace of V.