Math 224 **Exam #1, In-Class Portion**
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INSTRUCTIONS: Complete the following problems without the aid of any person or any technology more advanced than a pencil or pen.

1. (9 points) Find the set of solutions for the following system of equations.

$$\begin{array}{rrrl} 3x+ & 3y+ & 12z = & 6 \\ x+ & y+ & 4z = & 2 \\ 2x+ & 5y+ & 20z = & 10 \\ -x+ & 2y+ & 8z = & 4 \end{array}$$

2. (8 points) Let $A = \begin{bmatrix} 2 & 0 & -1 \\ 4 & -5 & 2 \end{bmatrix}$, let $B = \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & 3 \end{bmatrix}$, let $C = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$, and let $D = \begin{bmatrix} 3 & 5 \\ -1 & 4 \end{bmatrix}$.

Evaluate each of the following or state why the sum/product is not defined.

(a) $-2A$

(b) $B - 2A$

(c) AC

(d) CD

3. (8 points) Invert the matrix $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 0 \\ -2 & 0 & 3 \end{bmatrix}$ by any (correct) method, or show that it is not invertible.

4. (8 points) Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

(a) Express A^{-1} as a product of elementary matrices.

(b) Use your answer from the previous part to express A as a product of elementary matrices.

5. (8+2 points) Find $\begin{vmatrix} 6 & 3 & 2 & 4 & 0 \\ 4 & 2 & 3 & 2 & 0 \\ -8 & -5 & 6 & 7 & 1 \\ 3 & 0 & 0 & 0 & 0 \\ 9 & 0 & -4 & 1 & 0 \end{vmatrix}$ by any combination of cofactor expansion and row-reduction. What does your answer tell you about the invertibility of the matrix?

6. (6 points) Assume that $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 7$. Evaluate the following.

(a) $\begin{vmatrix} a & b & c \\ d & e & f \\ 5g & 5h & 5i \end{vmatrix}$

(b) $\begin{vmatrix} g & h & i \\ a & b & c \\ d & e & f \end{vmatrix}$

(c) $\begin{vmatrix} a & b & c \\ 2d+a & 2e+b & 2f+C \\ g & h & i \end{vmatrix}$

7. (6 points) Each of the following statements is not quite true. Add a word or two to each sentence to make it a true (and interesting) statement. (Don't just add "not" to each one, for example.) You may either insert the word(s) with the "$\wedge$" symbol or rewrite the sentence.

(a) The row-echelon form of a matrix is always the same, no matter what row operations were used to get there.

(b) If k is any constant, then multiplying a row of a matrix by k is an elementary row operation.

(c) Any linear system with fewer equations than unknowns has an infinite number of solutions.

8. (3 points) Prove that if AB is a square matrix, then the product BA is defined.

9. (4 points) Show that if $CA = I$ then $A\vec{x} = \vec{0}$ has only the trivial solution. Note that we are *not* assuming that A is square, so no results about invertibility may be used.

10. (4 points) Let A and P be square matrices of the same size, with P invertible. Show that $\det(A) = \det(PAP^{-1})$