

Final Exam
Part I: No Maple

Math 224: Linear Algebra

Name: _____

21 points

- You must show all work to receive full credit.
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1. (7 pts) Determine the general solution of

$$\begin{aligned}x + 3y + 2z &= 6 \\2x + y &= 3 \\2y + z &= 0\end{aligned}$$

using either the Gaussian Method or the Gauss-Jordan Method.

2. (7 pts) Calculate the inverse of

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{pmatrix}$$

provided it exists.

3. (7 pts) Use Cramer's Rule to calculate z for the system

$$\begin{aligned}x + 4z &= 2 \\2x + y - z &= 3 \\y + z &= 1.\end{aligned}$$

Final Exam

Part II: Maple may be used.

Math 224: Linear Algebra

Name: _____

79 points

- You must show all work to receive full credit.
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1. (10 pts) Determine if the set $H = \{(x, y, z) : x - 4y - z = 0\}$ is a subspace of \mathfrak{R}^3 .

2. (10 pts) Let $w_1 = \begin{pmatrix} 1 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $w_2 = \begin{pmatrix} 3 \\ 2 \\ 2 \\ 3 \end{pmatrix}$, $w_3 = \begin{pmatrix} 1 \\ 4 \\ 0 \\ 1 \end{pmatrix}$, and $w_4 = \begin{pmatrix} -1 \\ 3 \\ 1 \\ 2 \end{pmatrix}$. Find a basis for the $\text{Span}(w_1, w_2, w_3, w_4)$ and the dimension of the space.

3. (15 pts) Let

$$A = \begin{pmatrix} 1 & 2 & 1 & 3 \\ 2 & 1 & 2 & 1 \\ 1 & -1 & 0 & 1 \end{pmatrix}$$

a. Find a basis for the column space of A .

b. Find a basis for the row space of A .

c. Find a basis for the null space of A .

3. (5 pts) Let $A = \begin{pmatrix} 1 & -1 & 2 & 1 \\ 2 & -1 & -1 & 4 \\ -4 & 5 & -10 & -6 \\ 3 & -2 & 10 & -1 \end{pmatrix}$. Determine the rank(A) and the nullity(A).

4. (5 pts) Determine if the matrices $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$, $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ are linearly independent in M_{22} .

5. (9 pts) Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be given by $T \left(\begin{pmatrix} x \\ y \\ z \end{pmatrix} \right) = \begin{pmatrix} 2x - y \\ 2x + y \\ x + z \end{pmatrix}$.

a. Is T a linear transformation? If so, give the standard matrix representation of T .

b. Does T^{-1} exist. If so, give the standard matrix representation of T^{-1} .

6. (15 pts) Let $A = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 4 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 2 & 4 \end{pmatrix}$.

a. Give the characteristic polynomial of A .

b. Calculate the eigenvalues of A .

c. Determine the corresponding eigenspace for each eigenvalue.

d. Give the algebraic multiplicity and the geometric multiplicity of each eigenvalue.

7. (10 pts) Let $\mathbf{p} = p(x)$ and $\mathbf{q} = q(x)$ be polynomials in P_2 . Define

$$\langle \mathbf{p}, \mathbf{q} \rangle = p(0)q(0) + p(1)q(1) + p(2)q(2).$$

a. Show that $\langle \mathbf{p}, \mathbf{q} \rangle$ defines an inner product on P_2 .

b. Give a general formula for $\|\mathbf{p}\|$.