Exam 2

Math 224: Linear Algebra Name: ____________________

100 points 10/16/2001

- You must show all work to receive full credit.

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1. (10) Express the matrix $\begin{pmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$ as a product of elementary matrices. Show or explain how you arrived at your choice of elementary matrices.

2. (10) If A and B are appropriately sized symmetric matrices, determine if each of the following are symmetric.

(a) $A - 2B$

(b) A^T

(c) AB

(d) B^{-1}, provided it exists of course.

(d) $A^{-1}A$, provided A^{-1} exists of course.

3. (12) Find the determinants using the method of your choosing. (Some methods may be better than others.)

(a) $\begin{vmatrix} 2 & -12 & 8 \\ 2 & 5 & 9 \\ -1 & 6 & -2 \end{vmatrix}$

(b) $\begin{vmatrix} 2 & 4 & 2 & 0 \\ 7 & 6 & 7 & 4 \\ -3 & 0 & 0 & 0 \\ 1 & -2 & 1 & 9 \end{vmatrix}$

(c) $\begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 & 1 & 1 \\ 1 & 3 & 2 & 1 & 1 & 1 \\ 1 & 4 & 3 & 2 & 1 & 1 \\ 1 & 5 & 4 & 3 & 2 & 1 \\ 1 & 6 & 5 & 4 & 3 & 2 \end{vmatrix}$

4. (6) Consider the matrix $A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$ and assume that $|A| = 3$. Determine the following:

(a) $\begin{vmatrix} 3a & 3b & 3c \\ -d & -e & -f \\ 2g & 2h & 2i \end{vmatrix}$

(b) $\begin{vmatrix} a & g & d \\ b & h & e \\ c & i & f \end{vmatrix}$

(c) $\begin{vmatrix} a & b & c \\ 2d-a & 2e-b & 2f-c \\ g-3a & h-3b & i-3c \end{vmatrix}$

5. (12) Find the following determinants if A and B are $n \times n$ matrices such that $|A| = 5$ and $|B| = 3$.

(a) $|AB^2|$

(b) $|(A^{-1})(2B)|$

(c) $|(A^T B)^{-1}|$

(d) $|Adj(B)|$

6. (8) If $A^{-1} = A^T$, then find all possible values of $|A|$.

7. (12) Use Cramer's Rule to find y for the system

$$\begin{aligned} x - 2y - z &= 1 \\ x + z &= 4 \\ 2x - y - 2z &= 2 \end{aligned}$$

8. (10) Prove that if A, B, and P are $n \times n$ matrices with P invertible and $A = PBP^{-1}$ then $|A| = |B|$.

9. (20) Given $A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 0 & 5 \\ 1 & 0 & 0 & 7 \end{pmatrix}$ find the matrix of cofactors for A and A^{-1} if it exists.