

Exam 1

Math 224: Linear Algebra

Name: _____

100 points

09/25/2001

- You must show all work to receive full credit.
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1) (12 pts) Solve the following linear system using Gauss Jordan Elimination.

$$-x_1 + 2x_2 + x_3 - 2x_4 = -2$$

$$-x_1 + x_2 - 3x_3 + 3x_4 = 3$$

$$x_1 + x_2 - 3x_3 + x_4 = 1$$

2. (6 pts) Determine the values of h such that the matrix $\begin{pmatrix} 2 & -6 & -3 \\ -4 & 12 & h \end{pmatrix}$ is the augmented matrix of a consistent linear system.

3. (10 pts) Let $A = \begin{pmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$. For which values in matrix $B = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ does the system $AX = B$ have unique solutions.

4. (16 pts) Compute the products below. If a product is undefined, then state why it is undefined.

a. $\begin{pmatrix} 3 & 1 & 2 \\ 2 & 2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 3 & 1 \\ 0 & 1 \end{pmatrix}$

b. $(1 \ 1 \ 1) \begin{pmatrix} 2 & 3 \\ 3 & 4 \\ 5 & 1 \end{pmatrix}$

c. $\begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 2 \\ 2 & 2 & 1 \\ 4 & 0 & 1 \end{pmatrix}$

d. $\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$

5. (12 pts) Let $A = \begin{pmatrix} 1 & 0 & 2 \\ 3 & 1 & 7 \\ 0 & 1 & 4 \end{pmatrix}$. Calculate A^{-1} and verify that $A^{-1}A = I$.

6. (6 pts) Let $A = \begin{pmatrix} 1 & 1 & b \\ a & b & b - a \\ 1 & 1 & 0 \end{pmatrix}$. Find a value for a and b such that A is not invertible. Explain how you know it is not invertible.

7. (6 pts) Find x where: $\begin{pmatrix} x & 1 - x^2 \\ 2 & 2x \end{pmatrix}^{-1} = \begin{pmatrix} -1 & 1 \\ 1 & \frac{-1}{2} \end{pmatrix}$

8. (6 pts) Determine a general formula for A^n , where n is a positive integer, if $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

9. (14 pts) Complete each of the following properties appropriately.

a. $(A + B)^T =$

b. $(AB)^T =$

c. $(AB)^{-1} =$

d. $(A^T)^T =$

e. $(kA)^T =$

f. $(kA)^{-1} =$

g. $(A^T)^{-1} =$

10. (12 pts) State whether A is invertible or not provided the following condition hold. Give a reason why for each answer.

a. The reduced row-echelon form of the square matrix A has no zero rows.

b. $x = t$, $y = 2 + 3t$, and $z = 2 - t$ is the general solution of $AX = B$.

c. $A = \begin{pmatrix} 15 & 1 & 7 & 10 \\ 2 & 14 & 11 & 12 \\ 1 & 0 & 0 & 8 \end{pmatrix}$.

d. $AX = B$ for $B = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ is consistent if $b_1 - 3b_2 - b_3 = 0$.