Math 206 · Exam #2 Solutions

Hartlaub - Fall 2006.

1. a. C.I.s are always centered at $\bar{x}$, so $\bar{x} = \frac{114.4 + 115.6}{2} = 115$

b. More confident $\Rightarrow$ wider interval.

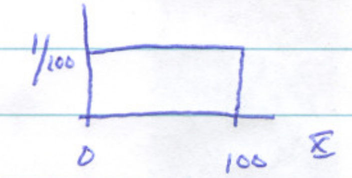
90% C.I. is $(114.4, 115.6)$; 99% C.I is $(114.1, 115.9)$

c. No. μ is fixed, so $P(114.4 \le \mu \le 115.6)$ is either 0 or 1.

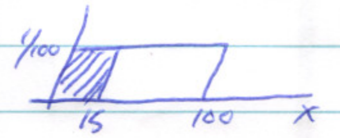
d. No. In the long run, <u>approximately</u> 99% of the C.I.s will contain μ. We should get close to 99 of the 100 intervals containing μ, but there is no reason to believe we will get exactly 99 out of 100.

2. a. X is a continuous R.V. that is uniformly distributed over the interval $(0, 100)$

b.



c. $P(X \le 15) = 15 \times \frac{1}{100} = \frac{15}{100} = .15$



d. $P(X \ge 40) = 60 \times 1/100 = \frac{60}{100} = .60$

e. $P(X = 75) = 0$

f. $E[X]$ = center of the probability density curve = 50. seconds.

3. Let X = # of mountain bikes purchased ; $X \sim B(n=16, p=.7)$

a. $P(X \le 10) \underset{MTB}{=} .340218$

b. $P(X < 10) = P(X \le 9) \underset{MTB}{=} .175313$

c. $P(X \ge 10) = 1 - P(X < 10) = 1 - P(X \le 9) \underset{MTB}{=} 1 - .175313 = .824687$

d. $P(X = 10) = \binom{16}{10}(.7)^{10}(.3)^{6} \underset{MTB}{=} .164904$

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#4. a. Since the sample size is small ($n=10$) and $np = 10(.3) = 3 \le 10$, the distribution of the sample proportion will be somewhat skewed (with center at $p=.3$) and the normal approximation should not be used.

b. $\mu_{\hat{p}} = p = .3$ $\quad \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.3(.7)}{400}} = .0229$

c. $P(.25 \le \hat{p} \le .35) = P\left(\frac{.25-.3}{.0229} \le Z \le \frac{.35-.3}{.0229}\right)$

$= P(-2.1834 \le Z \le 2.1834)$

$= .9855 - .0145 = .971$

d. The probability calculated in part (c) is smaller than would be the case if $n=500$. If $n=500$, then the distribution of $\hat{p}$ would be more concentrated about $p=.3$. Less variability about the center ($\mu_{\hat{p}}=.3$) would result in a higher probability of getting $\hat{p}$ between .25 and .35.

#5. Let X_i = amount of money spent by a customer

$\mu_{X_i} = 100 \quad \sigma_{X_i} = 30 \quad T = \sum_{i=1}^{50} X_i$

$P(T > 5300) = P\left(\bar{X} > \frac{5300}{50}\right)$

$= P(\bar{X} > 106)$

$= P\left(Z > \frac{106-100}{30/\sqrt{50}}\right)$ $\quad \sigma_{\bar{X}} = \frac{30}{\sqrt{50}} = 4.2426$

$= P(Z > 1.4142)$

$= 1 - .9213 = .0787$

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#6. a. $P(X=3 \cap Y=2) \underset{indep}{=} P(X=3) \times P(Y=2) = .3 \times .2 = .06$

b. $P(\text{same \# of cars and buses}) = P(X=Y)$

$= P(X=0 \cap Y=0) + P(X=1 \cap Y=1) + P(X=2 \cap Y=2)$

$= (.05)(.5) + (.1)(.3) + (.25)(.20)$

$= .025 + .03 + .05 = \boxed{.105}$

c. Total Amount Collected $= 3X + 10Y$

$\mu_{3X+10Y} = \mu_{3X} + \mu_{10Y} = 8.4 + 7 = \boxed{\$15.4}$

$\sigma^2_{3X+10Y} \underset{indep}{=} \sigma^2_{3X} + \sigma^2_{10Y} = 14.94 + 61.^{00} = \boxed{\$75.94}$

(see work below)

For cars (X): $\mu_X = 0(.05) + 1(.1) + 2(.25) + 3(.3) + 4(.2) + 5(.1) = 2.8$

$\sigma^2_X = (0-2.8)^2(.05) + (1-2.8)^2(.1) + (2-2.8)^2(.25) + (3-2.8)^2(.3)$
$+ (4-2.8)^2(.2) + (5-2.8)^2(.1) = 1.66$

Since the toll is \$3 for each car, we want the mean and variance of $3X$

$\mu_{3X} = 3\mu_X = 3(2.8) = \$8.4$

$\sigma^2_{3X} = 3^2\sigma_X^2 = 9(1.66) = \$14.94$

For buses (Y): $\mu_Y = 0(.5) + 1(.3) + 2(.2) = .7$

$\sigma^2_Y = (0-.7)^2(.5) + (1-.7)^2(.3) + (2-.7)^2(.2) = .61$

Since the toll is \$10 for each bus, we want the mean and variance of $10Y$

$\mu_{10Y} = 10\mu_Y = 10(.7) = \$7.^{00}$

$\sigma^2_{10Y} = 10^2\sigma^2_Y = 100(.61) = \$61.^{00}$

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\#7 Let μ = mean heat conductivity for this type of glass

$H_0: \mu = 1$

vs $H_1: \mu \neq 1$

(Stat > Basic Stats > 1-sample t)

Test Stat: $t = 8.95 = \frac{1.11818 - 1}{.0132}$

p-value $= P(T_{10} > 8.95 \text{ or } T_{10} < -8.95) \approx 0.00$

Conclusion: Since the p-value $\approx 0.00 < \alpha = .05$, we reject H_0 and conclude that the heat conductivity for this type of glass is significantly different from 1.

NOTE: The test above assumes that the data are from a normal population. The probability plot shows some clear departures from linear trend, so a sign test might be a better option in this setting.

Sign Test: $H_0: \eta = 1$ vs $H_1: \eta \neq 1$

(Stat > Nonparametrics > 1-Sample Sign)

All 11 values are above 1, so $B = 11$

p-value $= .001$

Since $.001 < .05$, we make the same conclusion with the sign test that we did with the t-test.

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[Extra Credit]

$H_0: \mu_{PSS} = \mu_{HLF}$

vs

$H_a: \mu_{PSS} \neq \mu_{HLF}$

Assume independent observations & normality. Normal probability plots are roughly linear, so normality assumption is OK. (Although it's hard to tell shape with only 5 obs.)

Test Stat: 2 sample $t = -3.33$

p-value $= .021$

Conclusion: Since $.021 < .05$, we reject H_0 and conclude that the 2 processes result in different mean breaking strengths.

Could use Mann-Whitney test for two medians.

$H_0: \eta_{PSS} = \eta_{HLF}$ vs $H_a: \eta_{PSS} \neq \eta_{HLF}$

Assume independence & same shape of distributions.

Test Stat: $W = 16$

p-value: $.0216$

Same conclusion as above.