The Pendulum[1]

In this lab you will study the motion of a pendulum that can swing in a full 360˚ circle. To keep the physical details as simple as possible, we will assume its mass is 1 kg. and that all of the mass is concentrated in the center of the pendulum bob, 1 meter from the pivot point. Assume that the pendulum is *x* meters from its rest position at time *t*, where *x* is measured around the circular path that the bob traces out. (Since the circle has radius 1 meter, this also corresponds to an angular displacement of *x* radians from the rest position!) Assume that the velocity is *v*. Take counterclockwise positions and velocities to be positive, clockwise ones to be negative. When the pendulum is at rest, we have $x = v = 0$. This is all shown in Figure 1.

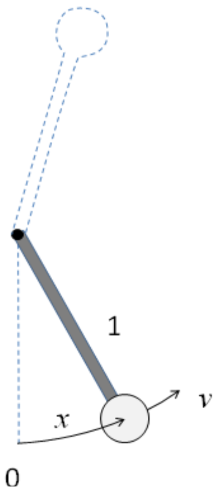

Figure 1

When the pendulum is moving, there must be forces at work. We will ignore friction. Thus the only force that acts on the pendulum is gravity. Gravity points straight down, as shown in Figure 2.

Building the Model:

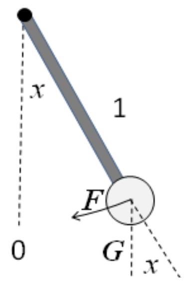

Figure 2

Question 1: What is the relationship between *x* and *v*?

Question 2: What is the relationship between ***F*** and ***G***?

Question 3: Using your answer to question 2, write a differential equation that describes the rate of change of *v* in terms of *x*.

Analyzing the Model:

NEED TO INCLUDE GUIDANCE HERE ABOUT SHOWING EULER'S METHOD ISN'T GOING TO WORK AND THEN GENERALIZING EULER'S METHOD.

Question 4: Assume an initial velocity of 0 m/sec and an initial angular displacement of $\pi/8$ radians. Analyze the model using Euler's method. Get a graph that shows the displacement and the velocity over time. (Note: Error in Euler's method is very sensitive to "turning corners" so periodic functions such as these require a large number of time-steps and some patience. Experiment a bit to see what works.)

---

[1] This lab is adapted from the book <u>Calculus in Context</u> by Callahan et. al. Some portions of the lab are taken verbatim from that source.

Question 5: Now keep the initial velocity constant, but consider different initial displacements between 0 and π. (Why π?) Get a series of graphs that show how the motion of the pendulum changes as you increase the initial angular displacement.

Question 6. Use the graphs that you obtained in question 5 to describe changes in the motion of the pendulum as the initial displacement increases towards π. Give a physical explanation for these changes.

Question 7. The initial velocity of the pendulum is sometimes called the "impulse." We have been looking at what happens when the impulse is 0. Now we want to consider what happens when the impulse is not 0. Suppose that we start with the pendulum straight down and give it a push (an impulse).

a) What is the period of the pendulum when the initial velocity is 1? What about 2?
b) By experimenting, find out how large an impulse you will need to get the pendulum to go over the top and start spinning. Of course any enormous value for v(0) will cause this to happen. Your task is to find the *threshold*; this is the smallest initial impulse that will cause spinning.