Math 333
Practice Problems: Higher Order Linear Differential Equations

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In each of the following problems, find the general solution of the given differential equation.

1. $y''' - y'' - y' + y = 0$

2. $2y''' - 4y'' - 2y' + 4y = 0$

3. $y^{(6)} + y = 0$

4. $y^{(6)} - 3y^{(4)} + 3y'' - y = 0$

5. $y^{(5)} - 3y^{(4)} + 3y''' - 3y'' + 2y' = 0$

6. $y^{(8)} + 8y^{(4)} + 16y = 0$

7. $y''' - 5y'' + 3y' + y = 0$

8. $18y''' + 21y'' + 14y' + 4y = 0$

9. $y''' - y'' - y' + y = 2e^{-2t} + 3$

10. $y^{(4)} + y''' = \sin(2t)$

11. $y^{(4)} + 2y'' + y = 3 + \cos(2t)$

Answers.

1. $y(t) = c_1 e^t + c_2 t e^t + c_3 e^{-t}$

2. $y(t) = c_1 e^t + c_2 e^{2t} + c_3 e^{-t}$

3. $y(t) = c_1 \cos t + c_2 \sin t + c_3 e^{\sqrt{3}t/2} \cos(\frac{1}{2}t) + c_4 e^{\sqrt{3}t/2} \sin(\frac{1}{2}t) + c_5 e^{-\sqrt{3}t/2} \cos(\frac{1}{2}t) + c_6 e^{-\sqrt{3}t/2} \sin(\frac{1}{2}t)$

4. $y(t) = c_1 e^t + c_2 t e^t + c_3 t^2 e^t + c_4 e^{-t} + c_5 t e^{-t} + c_6 t^2 e^{-t}$

5. $y(t) = c_1 + c_2 e^t + c_3 e^{2t} + c_4 \cos t + c_5 \sin t$

6. $y(t) = c_1 e^t \cos t + c_2 t e^t \cos t + c_3 e^t \sin t + c_4 t e^t \sin t + c_5 e^{-t} \cos t + c_6 t e^{-t} \cos t + c_7 e^{-t} \sin t + c_8 t e^{-t} \sin t$

7. $y(t) = c_1 e^t + c_2 e^{(2+\sqrt{5})t} + c_3 e^{(2-\sqrt{5})t}$

8. $y(t) = c_1 e^{-t/2} + c_2 e^{-t/3} \cos(t/sqrt3) + c_3 e^{-t/3} \sin(t/\sqrt{3})$

9. $y(t) = c_1 e^t + c_2 t e^t + c_3 e^{-t} + \dfrac{1}{2} t e^{-t} + 3$

10. $y(t) = c_1 + c_2 t + c_3 t^2 + c_4 e^{-t} + \dfrac{1}{20} \sin(2t) + \dfrac{1}{40} \cos(2t)$

11. $y(t) = c_1 \cos t + c_2 \sin t + c_3 t \cos t + c_4 t \sin t + 3 + \dfrac{1}{9} \cos(2t)$