

Math 333

Homework Solutions: Higher Order Linear Differential Equations

In each of the following problems, find the general solution of the given differential equation.

1. $y''' - 3y'' + 3y' - y = 0$

Solution. $y(t) = k_1e^t + k_2te^t + k_3t^2e^t$

2. $y^{(4)} - 5y'' + 4y = 0$

Solution. $y(t) = k_1e^t + k_2e^{2t} + k_3e^{-2t} + k_4e^{-t}$

3. $y^{(4)} - 8y' = 0$

Solution. $y(t) = k_1 + k_2t + k_3e^t + k_4e^{-t} + k_5 \cos t + k_6 \sin t$

4. $y''' + 5y'' + 6y' + 2y = 0$

Solution. $y(t) = k_1e^{-t} + k_2e^{(-2+\sqrt{2})t} + k_3e^{(-2-\sqrt{2})t}$

5. $y^{(4)} - 7y''' + 6y'' + 30y' - 36y = 0$

Solution. $y(t) = k_1e^{3t} + k_2e^{-2t} + k_3e^{(3+\sqrt{3})t} + k_4e^{(3-\sqrt{3})t}$

6. $y^{(4)} - y = 3t + \cos t$

Solution. $y(t) = k_1e^t + k_2e^{-t} + k_3 \cos t + k_4 \sin t - 3t - \frac{1}{4}t \sin t$

7. $y''' + y'' + y' + y = e^{-t} + 4t$

Solution. $y(t) = k_1e^{-t} + k_2 \cos t + k_3 \sin t + \frac{1}{2}te^{-t} + 4(t - 1)$

8. $y^{(4)} - 4y'' = t^2 + e^t$

Solution. $y(t) = k_1 + k_2t + k_3e^{-2t} + k_4e^{2t} - \frac{1}{3}e^t - \frac{1}{48}t^4 - \frac{1}{16}t^2$