Math 347

1.4 #9 Solution

(a) We will use the following variables in the model:

n	number of 10 cent price increases (in dollars per paper)
s	number of subscriptions (in papers)
P	profit (in dollars)

We have $s = 80000 - 5000n$ since it is estimated that the paper would lose 5,000 subscribers per 10 cent price increase. The profit is given by

$$P(n) = (80000 - 5000n)(1.50 + 0.10n).$$

To find the price that maximizes profit, we maximize the function P. See the Maple file news.mw for the calculations. The maximum profit occurs when $n = 0.5$, so we conclude that the subscription price that maximizes profit is \$1.55 (i.e. the company should raise the price by 0.5×0.10=\$0.05. The corresponding maximum profit is approximately \$120,125.

(b) Next we calculate the optimal subscription price with the parameter 5,000 replaced by 3,000, etc. We obtain the following results:

parameter	optimal subscription price	maximum profit
3000	2.08	130,210
4000	1.75	122500
5000	1.55	120125
6000	1.42	120410
7000	1.32	122220

(c) Next we calculate the sensitivity of the optimal subscription price to the number of subscribers lost when the subscription price increases by 10 cents. We let α denote the number of subscribers lost when the subscription price increases by 10 cents. For $\alpha = 5000$, we compute $S(p^\star, \alpha) = -0.52$, which means that if the number of lost subscriptions is 20% higher than expected, then the optimal price is about 10% lower. This is confirmed by the data observed in part (b).

(d) Based on the results of this model, the newspaper should not make a change in its subscription price. The chart in part (b) shows that for the current estimate of 5,000 lost subscribers per 10 cent price increase, we are already very close to the optimal subscription price. If we did raise the price by 5 cents to the optimal price of \$1.55, then we would only increase our estimated revenue by about \$125, or 0.10%.