Math 112
Monday, January 14, 2007
l'Hôpital's Rule

---

l'Hôpital's rule can be used to compute limits containing indeterminate forms. In its most basic form, l'Hôpital's rule states the following. Consider the limit

$$\lim_{x\to a}\frac{f(x)}{g(x)}.$$

Suppose that

$$\lim_{x\to a} f(x) = \pm\infty \text{ AND } \lim_{x\to a} g(x) = \pm\infty$$

OR

$$\lim_{x\to a} f(x) = 0 \text{ AND } \lim_{x\to a} g(x) = 0.$$

Then

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)},$$

assuming that the limit on the right-hand side exists.

Examples.

1. Compute $\lim_{x\to\infty}\frac{x}{e^x}$.

2. Compute $\lim_{x\to 0}\frac{x}{\sin x}$.

3. Compute $\lim_{x\to 0}\frac{e^x-1}{x}$.

4. Compute $\lim_{x\to 0^+} x\ln x$.

5. Compute $\lim_{x\to 0}\frac{\sin^2 x}{\cos x - 1}$.

6. Compute $\lim_{x\to\infty}\frac{x^2}{2^x}$.

7. Compute $\lim_{x\to\infty}(1+\frac{1}{x})^x$.

8. Suppose that f and g are differentiable functions and that $g(0)\neq 0$. Show that

$$\lim_{x\to 0}\frac{xf(x)}{(e^x-1)g(x)} = \frac{f(0)}{g(0)}.$$

9. **Challenge.** Compute $\lim_{x\to\ 0}\frac{1}{x}\int_0^x (1+\sin(2t))^{\frac{1}{t}}\,dt$.