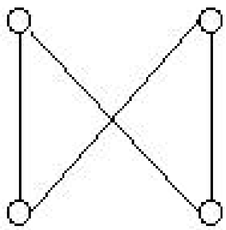
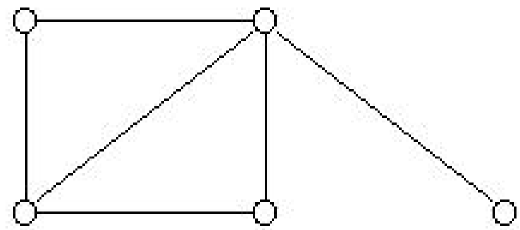
Math 347
Hamilton Cycles Problems

1. Construct a graph with a Hamilton cycle but no Euler cycle.
2. Construct a graph with an Euler cycle but no Hamilton cycle.
3. Determine which of the following graphs are Hamiltonian.

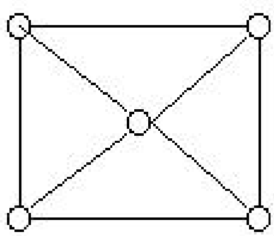
G1:



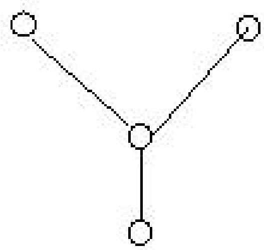
G2:



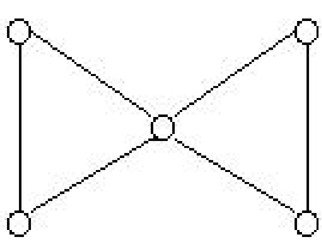
G3:



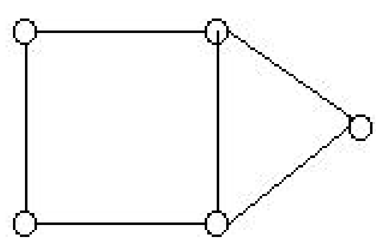
G4:



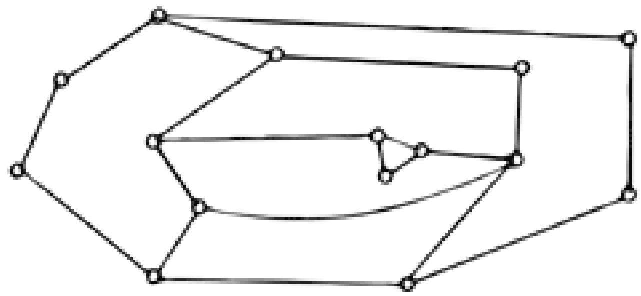
G5:



G6:



4. Prove that there is no Hamiltonian cycle in the following graph.



5. When pictures are transmitted electronically (for example, from spacecraft imaging systems to earth), the pictures are transmitted as a long sequence of numbers in which each number represents a darkness value for one of the pixels in the picture. For simplicity, assume that the darkness values range between 1 and 8. These numbers are sent as a sequence of 0's and 1's. A straightforward encoding scheme might be to express each number in its binary representation (e.g. 1 as 001, 2 as 010, 3 as 011, etc.).

   However, a better scheme, called a *Gray code*, uses an encoding property with the property that *two consecutive numbers are encoded by binary sequences that are almost the same, differing in just one position.* For example, a fragment of a Gray code might be 4 as 010, 5 as 011, 6 as 001. The advantage of such an encoding is that if a transmission error causes one binary digit in a sequence to be misread, then the mistaken sequence will often be interpreted as a darkness number that is almost the same as the true darkness number. For example, in the preceding fragment of a Gray code, if 011 (5) were transmitted an an error in the last position caused 010 (4) to be received, the resulting small change in darkness would not seriously affect the picture.

   We can translate the problem of finding a Gray code for the 8 darkness numbers into the problem of finding a Hamilton cycle in a graph. Define the graph as follows. Each vertex corresponds to a 3-digit binary sequence (i.e. a 3 digit sequence of 0's and/or 1's), and two vertices are connected with an edge if their binary sequences differ in just one place.

   Construct this graph, and argue that it must contain a Hamilton cycle (and hence, a Hamilton path). A Hamilton path is just a Hamilton cycle with the final vertex omitted (i.e. you do not revisit the starting vertex). So a Hamilton path is a path that visits each vertex exactly once. Then, argue that the order in which vertices (binary sequences) occur along a Hamilton path produces a Gray code That is, 1 is encoded as the first binary sequence in a Hamilton cycle, 2 is encoded as the second, and so on. Construct such a Hamilton cycle, and give the resulting Gray code.