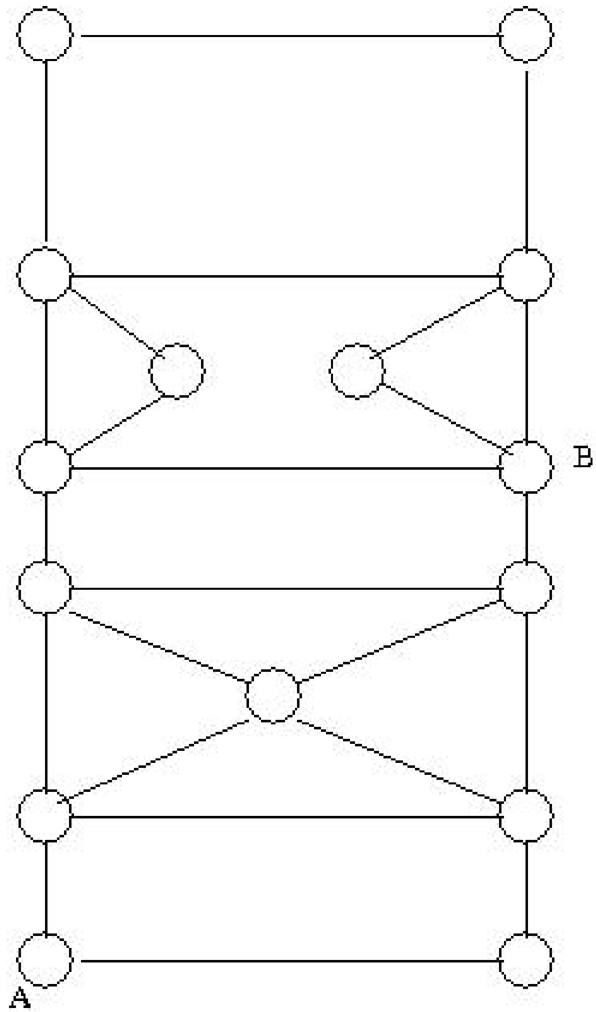
Math 347
Euler Cycles Problems

1. Suppose that a new artificial island were placed in the Pregel River in between islands B and C. If the bridge from B to C is removed and several bridges built connecting the new island to all other bodies of land (any number of bridges to other bodies of land allowed), is it possible to find a walk that crosses each bridge exactly once? If so, find such a walk.

2. A **bridge** is an edge whose removal disconnects a graph. Is it possible for a graph with an Euler cycle to contain a bridge? Prove or give a counterexample.

3. A **trail** is a sequence of consecutively linked edges in which no edge can appear more than once. Note that a cycle is a trail, but a trail can start and end at different vertices. An **Euler trail** is a trail that contains all the edges in a graph (and visits each vertex at least once). Construct an example of a graph (with at least 6 vertices, just to make it interesting) that contains an Euler trail but not an Euler cycle, and an example of a graph that contains neither an Euler trail nor an Euler cycle. Can you make a conjecture for a criterion for a graph G to have an Euler trail but not an Euler cycle?

4. A highway inspector must periodically drive along the several highways shown schematically in the figure below and inspect the roads for debris and possible repairs. If the inspector lives in town A, is it possible to find a round trip, beginning and ending at A, which takes her over each section of highway exactly once? If so, construct such a trip. If the inspector lives in town B, is such a trip possible? If so, construct it.



5. A **directed graph** is a graph in which the edges are directed from a particular vertex to another vertex (i.e. edges are not necessarily two-way edges). Try to make a conjecture for a criterion for a directed graph G to have an Euler cycle. Sketch a few directed graphs, and determine whether or not your directed graphs have Euler cycles.

6. A set of 8 binary digits (0 or 1) are equally spaced about the edge of a disk. We want to choose the digits so that they form a circular sequence in which every subsequence of length three is different. Model this problem with a graph with 4 vertices, one for each different subsequence of two binary digits. Make a **directed edge** for each subsequence of three digits whose origin is the vertex with the first two digits of the edge's subsequence and whose terminal vertex is the vertex with the last two digits of the edge's subsequence. Show that an Euler cycle (which does exist for this graph) will correspond to the desired 8-digit circular sequence. Find such an 8-digit circular sequence using your graph. Note that since your graph is directed, you can only travel along edges in the correct direction.