Blue and fin whale populations [MM 2.4.1]

Ecologists use the following model to represent the growth rates of two competing species, x, and y:

$$\begin{aligned} \frac{dx}{dt} &= r_1 x\left(1-\tfrac{x}{K_1}\right) - \alpha_1 xy \\ \frac{dy}{dt} &= r_2 y\left(1-\tfrac{y}{K_2}\right) - \alpha_2 xy \end{aligned}.$$

The variables x and y represent the number in each population, and $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are measured in units of whales per year. The parameters r_i represent the intrinsic growth rates of each species; K_i represents the maximum sustainable population in the absence of competition; and α_i represents the effects of competition. Studies of the blue whale and fin whale populations have determined the following parameter values (t in years):

	Blue whale	Fin whale
r	0.05	0.08
K	150,000	400,000
α	10^{-8}	10^{-8}

1. Determine the population levels x and y that maximize the total number of new whales born each year.
2. Examine the sensitivity of the optimal population levels to the intrinsic growth rates r_1 and r_2. Comment ecologically on the results.
3. Examine the sensitivity of the optimal population levels to the environmental carrying capacities K_1 and K_2. Comment ecologically on the results.
4. Assuming that $\alpha_1 = \alpha_2 = \alpha$, is it ever optimal for one species to become extinct? Can you think of an environmental/ecological situation in which it might be likely that $\alpha_1 = \alpha_2$? Can you think of a situation in which it might be unlikely for this to be true?