Volumes

Cross Sections

Cross sections perpendicular to the x-axis. Suppose that a solid has its base on the xy-plane between the vertical lines $x = a$ and $x = b$. For all x in $[a, b]$, let $A(x)$ denote the area of the **cross-section** at x perpendicular to the x-axis. If $A(x)$ is a continuous function, then the volume of the solid is given by:

$$V = \int_a^b A(x)\,dx$$

Cross sections perpendicular to the y-axis. Suppose that a solid has its base on the xy-plane between the horizontal lines $y = c$ and $y = d$. For all y in $[c, d]$, let $A(y)$ denote the area of the **cross-section** at y perpendicular to the y-axis. If $A(y)$ is a continuous function, then the volume of the solid is given by:

$$V = \int_c^d A(y)\,dy$$

Examples.

1. The base of a solid is the circle $x^2 + y^2 = 1$. Cross-sections of the solid perpendicular to the x-axis are squares with one side in the xy-plane. Find the volume of the solid.

2. The base of a solid is the region enclosed by the parabola $y = x^2$ and the line $y = 4$. Cross-sections of the solid perpendicular to the y-axis are semi-circles with diameter on the xy-plane. Find the volume of the solid.