Homework 12: Taylor and Maclaurin Series

1. Find the Taylor series for $f(x) = \cos x$ centered at $x = \pi/2$.
2. Find the Maclaurin series for $f(x) = \sin(x^4)$. You may use either the direct method (definition of a Maclaurin series) or a known Maclaurin series.
3. Find the Maclaurin series for $f(x) = e^{-3x}$. You may use either the direct method (definition of a Maclaurin series) or a known Maclaurin series.
4. Find the Maclaurin series for $f(x) = xe^{2x}$. You may use either the direct method (definition of a Maclaurin series) or a known Maclaurin series.
5. Find the Maclaurin series for $f(x) = x^3 \cos(x^2)$. You may use either the direct method (definition of a Maclaurin series) or a known Maclaurin series.
6. Find the Maclaurin series for $f(x) = \dfrac{e^x - 1}{x}$. You may use either the direct method (definition of a Maclaurin series) or a known Maclaurin series.
7. Use the series obtained in the previous problem to evaluate the indefinite integral $\displaystyle\int \frac{e^x - 1}{x}\,dx$.
8. Find the sum of the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{3^{2n}(2n)!}$.
9. Find the sum of the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n e^n}{n!}$.
10. The graph of $f(x)$ is shown below. Explain why the series

$$2.8 + 0.5(x-2) + 1.5(x-2)^2 - 0.1(x-2)^3 + \cdots$$

is *not* the Taylor series of f centered at $x = 2$.