A Quick Review of Integration

Area Definition.

$$\int_a^b f(x)\, dx = (\text{area above x-axis}) \quad - \quad (\text{area below x-axis})$$

We'll talk more about the area definition of the definite integral later in the course.

Fundamental Theorem of Calculus, Part 2.

$$\int_a^b f(x)\, dx = F(b) - F(a),$$

where $F(x)$ is *any anti-derivative* of $f(x)$, i.e. $F(x)$ is any anti-derivative of $f(x)$, i.e. any function such that $F'(x) = f(x)$.

Indefinite Integrals (also called Anti-derivatives).

$$\int f(x)\, dx = F(x) + C,$$

where $F(x)$ is any anti-derivative of $f(x)$.

Examples

1. Find the area between the graph of $f(x) = x^2$ and the x-axis over the interval $[0, 4]$.

2. Show that the area under one arch of the sine curve $y = \sin x$ is 2.

3. Evaluate $\int \frac{3}{x+1}\, dx$.

4. Evaluate $\int_0^\pi (1 + \cos x)\, dx$.

5. Evaluate $\int \frac{2}{x^2}\, dx$.

6. Evaluate $\int e^{-3x}\, dx$.

Integration by Substitution

Evaluate the following integrals.

1. $\int 2\cos{(2x + 5)}\, dx$

2. $\int \sqrt{3x - 5}\, dx$

3. $\int -2x\sqrt[3]{1 - x^2}\, dx$

4. $\int \tan^2 x \sec^2 x \, dx$

5. $\int \tan x \, dx$

6. $\int \frac{1}{x^2} \cos \frac{1}{x} \, dx$

7. $\int_0^2 4x^3 \sqrt{x^4 + 7} \, dx$

8. $\int_\pi^{2\pi} 2x \cos x^2 \, dx$