Math 347

Note: These problems are due on Tuesday, October 23 (in addition to the predator-prey problems distributed in class).

For Exercises 1-3, consider the two systems below:

System 1:

$$\begin{aligned} \frac{dx}{dt} &= 10x(1-\frac{x}{10})-20xy \\ \frac{dy}{dt} &= -5y+\frac{xy}{20} \end{aligned}$$

System 2:

$$\begin{aligned} \frac{dx}{dt} &= 0.3x-\frac{xy}{100} \\ \frac{dy}{dt} &= 15y(1-\frac{y}{15})+25xy \end{aligned}$$

1. In one of these systems, the prey are very large animals and the predators are small animals, such as mosquitoes and elephants. Thus it takes many predators to eat one prey, but each prey eaten is a tremendous benefit for the predator population. The other system has very large predators and very small prey. Determine which system is which and provide a justification for your answer.
2. For each system, describe the behavior of each population if the predators are extinct at time $t = 0$.
3. For each system, describe the behavior of each population if the prey are extinct at time $t = 0$.

For Exercises 4-7, consider the following predator-prey system (the same system as in part 1 of this homework):

$$\begin{aligned} \frac{dR}{dt} &= 2R-1.2RF \\ \frac{dF}{dt} &= -F+0.9RF \end{aligned}$$

4. Suppose the predators discover a second, unlimited source of food, but they still prefer to eat prey when they catch them. How would you modify this system to include this assumption? Construct phase plots and $R-t$ and $F-t$ plots with the initial condition $R(0) = 1$, $F(0) = 0.5$ for this situation.
5. Suppose the predators found a second food source that is limited in supply. How would you modify this system to include this fact? Construct phase plots and $R-t$ and $F-t$ plots with the initial condition $R(0) = 1$, $F(0) = 0.5$ for this situation.

6. Suppose predators migrate to an area if there are five times as many prey as predators in that area (i.e. if $R > 5F$) and they move away if there are fewer than 5 times as many prey as predators. How would you modify this system to take this into account? Construct phase plots and $R - t$ and $F - t$ plots with the initial condition $R(0) = 1$, $F(0) = 0.5$ for this situation.

7. Suppose prey move out of an area at a rate proportional to the number of predators in the area. How would you modify this system to take this into account? Construct phase plots and $R - t$ and $F - t$ plots with the initial condition $R(0) = 1$, $F(0) = 0.5$ for this situation.