Sequences Practice Problems

For each of the following problems, determine whether the sequence $\{a_n\}$ converges or diverges. If the sequence converges, find its limit.

1. $a_n = \dfrac{3+5n^2}{n+n^2}$
2. $a_n = \dfrac{n+1}{3n-1}$
3. $a_n = \dfrac{2^n}{3^{n+1}}$
4. $a_n = \dfrac{\sqrt{n}}{1+\sqrt{n}}$
5. $a_n = \dfrac{(n+2)!}{n!}$
6. $a_n = \dfrac{n}{1+\sqrt{n}}$
7. $a_n = \dfrac{(-1)^{n-1}n}{1+n^2}$
8. $a_n = \dfrac{(-1)^n n^3}{n^3+2n^2+1}$
9. $a_n = \dfrac{e^n+e^{-n}}{e^{2n}-1}$
10. $a_n = \cos\left(\dfrac{2}{n}\right)$
11. $a_n = n^2 e^{-n}$
12. $a_n = \dfrac{\cos^2 n}{2^n}$
13. $a_n = n\cos n\pi$
14. $a_n = \dfrac{\sin(2n)}{1+\sqrt{n}}$
15. $a_n = \dfrac{(\ln n)^2}{n}$
16. $a_n = (\ln(2n^2+1) - \ln(n^2+1))$
17. $a_n = \dfrac{(-1)^n n}{n+1}$

18. $a_n = 2 + \left(\dfrac{-2}{\pi}\right)^n$

19. (a) Consider the sequence $\{a_n\}$ defined by

$$a_1 = 1, \quad a_{n+1} = 4 - a_n$$

for $n \geq 1$. Is the sequence convergent or divergent?

(b) What happens if the first term is $a_1 = 2$?

20. Consider the sequence defined by

$$a_1 = 2, \quad a_{n+1} = \frac{1}{3 - a_n}$$

for $n \geq 1$. Use the Monotonic Sequence Theorem to show that the sequence is convergent.