Sequences Homework Part 2.

Solutions.

1. $a_n = \dfrac{\sin^2 n}{n^2+1}$

$$0 \leq \frac{\sin^2 n}{n^2+1} \leq \frac{1}{n^2+1}$$

$$\lim_{n\to\infty} 0 = 0 \qquad \lim_{n\to\infty} \frac{1}{n^2+1} = 0.$$

Thus, by the Squeeze Theorem,

$$\lim_{n\to\infty} \frac{\sin^2 n}{n^2+1} = 0.$$

Thus the sequence $\{a_n\}$ defined by $a_n = \dfrac{\sin^2 n}{n^2+1}$

converges to 0.

2. $a_n = \dfrac{n \cos n}{n^2+1}$

$$\frac{-n}{n^2+1} \leq \frac{n\cos n}{n^2+1} \leq \frac{n}{n^2+1}$$

$$\lim_{n\to\infty} \frac{-n}{n^2+1} = 0. \quad \lim_{n\to\infty} \frac{n}{n^2+1} = 0.$$

Thus, by the Squeeze Theorem,

$$\lim_{n\to\infty} \frac{n\cos n}{n^2+1} = 0.$$

Thus the sequence $\{a_n\}$ defined by

$a_n = \dfrac{n\cos n}{n^2+1}$ converges to 0.

3. $a_n = \left(1 + \frac{3}{n}\right)^{4n}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \left(1 + \frac{3}{n}\right)^{4n} = L$$

$$\ln L = \lim_{n\to\infty} \ln \left(1 + \frac{3}{n}\right)^{4n}$$

$$\ln L = \lim_{n\to\infty} 4n \cdot \ln\left(1 + \frac{3}{n}\right) = 4 \cdot \lim_{n\to\infty} \frac{\ln\left(1 + \frac{3}{n}\right)}{\frac{1}{n}}$$

$$\ln L \overset{L}{=} 4 \cdot \lim_{n\to\infty} \frac{\frac{1}{1+\frac{3}{n}} \cdot -3n^{-2}}{-n^{-2}}$$

$$\ln L = 4 \cdot 3 = 12 \Rightarrow L = e^{12}$$

The sequence converges to e^{12}