Separable Differential Equations

For certain types of differential equations, all x terms can be collected with dx and all y terms with dy, and a solution can be obtained by integrating both sides. Such equations are called **separable**, and the solution procedure is called separation of variables. More formally, a separable differential equation is one that can be written in the form

$$G(y)\,dy = F(x)\,dx.$$

Examples.

1. Determine whether each of the following differential equations is separable:
   (a) $y' = x^2 \cos y$
   (b) $\frac{dy}{dt} = \frac{\sin t}{y}$
   (c) $\frac{dy}{dt} = y + t$
2. Find the general solution of the differential equation $y' = \frac{x^2+2}{3y^2}$.
3. Find the solution of the initial-value problem $y' = \frac{3x}{2y}$, $y(1) = 4$.
4. (a) Find the general solution of the differential equation $y' = \frac{x^2+2}{1+4y+y^2}$.
   (b) Find the solution of the initial-value problem $y' = \frac{x^2+2}{1+4y+y^2}$, $y(0) = 1$.
5. Find the general solution of the differential equation $x^2 \frac{dy}{dx} = y$.
6. Find the solution of the initial-value problem $\frac{dy}{dt} = \frac{\sin t}{y}$, $y(0) = 3$.
7. Find the general solution of the differential equation $\frac{dy}{dx} = -\frac{e^x(y^2-1)}{y}$.
8. Find the general solution of the differential equation $y' = \frac{x^2+2}{3y^2}$.
9. Let $P(t)$ denote the population of a particular species at time t. Let C denote the **environmental carrying capacity** of the population. The **logistic population growth model** states that the rate of change of the population with respect to time is proportional both to the population itself and to the difference between the environment's carrying capacity and the population. Express this model symbolically using a differential equation. Let $P(0) = P_0$ denote the initial population. Solve the differential equation.