Volumes of Solids of Revolution

- **Disk Method**: Representative rectangle is *perpendicular* to the axis of revolution. R is the radius of a representative disk.

  1. Horizontal Axis of Revolution

  $$V = \pi \int_a^b [R(x)]^2 \, dx$$

  2. Vertical Axis of Revolution

  $$V = \pi \int_c^d [R(y)]^2 \, dy$$

- **Washer Method**: Representative rectangle is *perpendicular* to the axis of revolution. R is the outer radius of a representative washer and r is the inner radius of a representative washer.

  1. Horizontal Axis of Revolution

  $$V = \pi \int_a^b \left([R(x)]^2 - [r(x)]^2 \right) dx$$

  2. Vertical Axis of Revolution

  $$V = \pi \int_c^d \left([R(y)]^2 - [r(y)]^2 \right) dy$$

Examples.

1. Find the volume of the solid formed when the region bounded by $y = x^2$, $x = 0$, and $x = 1$ is revolved about the x-axis.

2. The region between the graph of $f(x) = \sin x \cos x$ and the x-axis over the interval $[0, \pi/2]$ is revolved about the x-axis. Find the volume of the resulting solid.

3. Find the volume of the solid formed when the region bounded by $y = x^2$, $y = 1$, and the line $x = 0$ is revolved about the y-axis.

4. The region in the first quadrant enclosed by the y-axis and the graphs of $y = \cos x$ and $y = \sin x$ is revolved about the x-axis. Find the volume of the resulting solid.

5. The region enclosed by the curves $y = x$ and $y = x^2$ is revolved about the x-axis. Find the volume of the resulting solid.

6. The region enclosed by the curves $y = x$ and $y = x^2$ is revolved about the line $y = 2$. Find the volume of the resulting solid.

7. The region enclosed by the curves $y = x$ and $y = x^2$ is revolved about the line $x = -1$. Find the volume of the resulting solid.