Math 333
Tuesday, February 26, 2008
Practice Problems: Reduction of Order

For each of the following differential equations, a known solution y_1 is given. Use the method of **reduction of order** to find a second solution of the given differential equation. Find the general solution $y(t)$ of the differential equation. Verify that $y(t)$ is indeed the general solution by showing that $y(t)$ is a linear combination of two solutions y_1 and y_2 whose Wronskian is nonzero.

1. $t^2y'' - 4ty' + 6y = 0$, $t > 0$; $y_1(t) = t^2$
2. $t^2y'' + 2ty' - 2y = 0$, $t > 0$; $y_1(t) = t$
3. $t^2y'' + 3ty' + y = 0$, $t > 0$; $y_1(t) = t^{-1}$
4. $t^2y'' - t(t+2)y' + (t+2)y = 0$, $t > 0$, $y_1(t) = t$
5. $xy'' - y' + 4x^3y = 0$, $x > 0$; $y_1(x) = \sin(x^2)$
6. $(x-1)y'' - xy' + y = 0$, $x > 1$; $y_1(x) = e^x$
7. $x^2y'' - (x - 0.1875y) = 0$, $x > 0$; $y_1(x) = x^{1/4}e^{2\sqrt{x}}$
8. $x^2y'' + xy' + (x^2 - 0.25)y = 0$, $x > 0$; $y_1(x) = x^{-1/2}\sin x$

Answers to the reduction of order practice problems.

1. $y_2(t) = t^3$
2. $y_2(t) = t^{-2}$
3. $y_2(t) = t^{-1} \ln t$
4. $y_2(t) = te^t$
5. $y_2(x) = \cos(x^2)$
6. $y_2(x) = x$
7. $y_2(x) = x^{1/4} e^{-2\sqrt{x}}$
8. $y_2(x) = x^{-1/2} \cos x$