Math 347
Tuesday, October 16, 2007
Lotka-Volterra Predator-Prey Model

So far, we have considered *competing species* models in which two competing species live in the same environment and compete for food and other ecological resources.

Next, we shall consider a *predator-prey* model in which one species eats another. The model that we shall study was developed independently by the Italian mathematician Vito Volterra and by the American biologist A.J. Lotka; hence, the dynamical system model for the predator-prey interaction of two species is called the Lotka-Volterra model.

Denote the predator population at time t by $x_1(t)$, and the prey population at time t by $x_2(t)$. The Lotka-Volterra predator-prey model is the dynamical system

$$\begin{aligned}\frac{dx_1}{dt} &= -r_1x_1 + \alpha_1x_1x_2 \\ \frac{dx_2}{dt} &= r_2x_2 - \alpha_2x_1x_2\end{aligned}$$

where the constants r_1, r_2, α_1, and α_2 are positive.

1. Interpret the terms of the dynamical system above in terms of predator-prey interactions.
2. Consider the following predator-prey dynamical system, where $R(t)$ denotes the number of rabbits (in thousands) at time t, and $F(t)$ denotes the number of foxes (in thousands) at time t:

$$\begin{aligned}\frac{dR}{dt} &= 2R - 1.2RF \\ \frac{dF}{dt} &= -F + 0.9RF\end{aligned}$$

   (a) What are the equilibrium points of this system?
   (b) Construct phase plots (solution curves F vs. R) for the initial condition $R(0) = 1$, $F(0) = 0.5$, and interpret your results. Create additional phase plots for various initial conditions, and describe your results.
   (c) Next construct plots of F vs. t and R vs. t for the initial condition $R(0) = 1$, $F(0) = 0.5$. Superimpose your plots on the same graph, and interpret the results. See the posted Maple (PredPrey.mw) file for the necessary Maple commands to do this.
   (d) Next, consider a modification of the predator-prey model above in which we assume that in the absence of predators, the prey population obeys a logistic rather than an exponential growth model. In particular, suppose that the environmental carrying capacity of the rabbit population is 2 (all other constants are assumed to be the same). Repeat parts (a), (b), and (c) for this modified system, and discuss the results.

(e) What happens if we account for harvesting? Assume that we have constant-effort harvesting in which the amount of each species caught per unit time is proportional to the population. Let H_R and H_F denote the harvesting coefficients for each species. Construct phase plots for different values of $H = H_R = H_F$ (between, say, 0 and 4), and interpret your results. Can you make any general statements about the effect of constant-effort harvesting on the equilibrium points?