Math 112
Why Does Integration by Parts Work?

Why does the integration by parts formula work? Suppose that u and v are both functions of x. Then, using the product rule,
$$(uv)' = uv' + u'v.$$
Equivalently, we can say that the product uv is an *anti-derivative* of $uv' + u'v$. Thus

$$
\begin{aligned}
u(x)v(x) &= \int \left(u(x)v'(x) + u'(x)v(x) \right) dx \\
&= \int u(x)v'(x)\, dx + \int v(x)u'(x)\, dx \\
&= \int u(x)\frac{dv}{dx}\, dx + \int v(x)\frac{du}{dx}\, dx \\
&= \int u(x)\, dv + \int v(x)\, du
\end{aligned}
$$

Thus we have:
$$uv = \int dv + \int v\, du.$$
Rearranging terms, we obtain the integration by parts formula:
$$\int u\, dv = uv - \int v\, du.$$