Math 112
Partial Fractions

When integrating a rational function whose numerator has a smaller degree than the denominator, we write the given rational function as a sum of its partial fractions.

- If the degree in the numerator is the same as the degree in the numerator or higher, we take the preliminary step of performing a long (Euclidean) division.
- If the denominator has more than two linear factors, we include a term corresponding to each factor. **Note: this will be the only type of partial fractions problem on the Gateway Exam.**

  As an example,
  $$\frac{x}{(x+2)(x+1)} = \frac{A}{x+2} + \frac{B}{x+1}.$$
- If a linear factor is repeated, we include extra terms in the partial fraction expression.

  As an example,
  $$\frac{x}{(x+2)^2(x-1)} = \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{x-1}.$$
- If the denominator is an irreducible quadratic then the corresponding partial fraction is of the form $\frac{Ax+B}{ax^2+bx+c}$. As an example,
  $$\frac{1}{(x^2+1)(x+2)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+2}.$$

Examples.

1. $\displaystyle\int \frac{5x-3}{(x-3)(x+1)}\,dx$

2. $\displaystyle\int \frac{2x+1}{(x-2)(x+5)}\,dx$

3. $\displaystyle\int \frac{2}{x^2+3x-4}\,dx$

4. $\displaystyle\int \frac{6x+7}{(x+2)^2}$

5. $\displaystyle\int \frac{x-1}{x^3+x^2}\,dx$

6. (Challenge) $\displaystyle\int \frac{10}{(x-1)(x^2+9)}\,dx$