Newton's Law of Cooling

A hot object cools down at a rate that depends on the temperature difference between the object and its environment–the greater the temperature difference, the faster the cooling occurs.

Newton's law of cooling states the following: The rate of change of the temperature of an object with respect to time is proportional to the difference between the object and its environment.

1. Let $T(t)$ denote the temperature of the object at time t, and let T_e denote the temperature of the surrounding environment. Let $T(0) = T_0$ denote the initial temperature of the object. Write a differential equation that expresses Newton's law of cooling, and solve the differential equation to find T as a function of t.

2. Suppose that a cup of coffee is placed in a 65-degree Fahrenheit room. After 2 minutes, the coffee has cooled from 190 degrees to 180 degrees. How long does the coffee stay above 100 degrees?

3. Experimentally, it has been determined that the constant of proportionality k in Newton's law of cooling is $k = -0.05$ for foam and $k = -0.08$ for cardboard. Suppose that two cups of 90 degrees Celsius coffee, one in a foam cup and one in a cardboard cup, are taken outside into 0 degrees Celsius weather. How long does the coffee in the foam cup stay above 70 degrees? How about the coffee in the cardboard cup? How hot is each cup after 5 minutes?