Introduction to Series Practice Problems

1. Let $a_n = \dfrac{2n}{3n+1}$.
   (a) Determine whether the sequence $\{a_n\}$ converges or diverges.
   (b) Determine whether the series $\displaystyle\sum_{n=1}^{\infty} a_n$ converges or diverges.

2. Show that the series
$$\sum_{n=1}^{\infty} \frac{n^2}{5n^2+4}$$
diverges.

3. Show that the series
$$\sum_{n=1}^{\infty} \frac{n+1}{2n-3}$$
diverges.

4. Show that the series
$$\sum_{k=1}^{\infty} \frac{k(k+2)}{(k+3)^2}$$
diverges.

5. Show that the series
$$\sum_{n=2}^{\infty} \frac{2}{n^2-1}$$
converges, and find the sum.

6. Show that the series
$$\sum_{n=1}^{\infty} \frac{2}{n^2+4n+3}$$
converges, and find the sum.

7. Show that the series
$$\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$$
converges, and find the sum.

8. Suppose that the n-th partial sum of a series $\sum_{n=1}^{\infty} a_n$ is $S_n = \frac{n-1}{n+1}$. Find a_n and $\sum_{n=1}^{\infty} a_n$.

9. Suppose that $\sum_{n=1}^{\infty} a_n$ $(a_n \neq 0)$ is known to converge. Explain why $\sum_{n=1}^{\infty} \frac{1}{a_n}$ diverges.