Answers: Practice with Improper Integrals

Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

1. $\displaystyle\int_1^\infty \frac{1}{(3x+1)^2}\,dx$ converges to $1/12$
2. $\displaystyle\int_{-\infty}^0 \frac{1}{2x-5}\,dx$ diverges
3. $\displaystyle\int_{-\infty}^{-1} \frac{1}{\sqrt{2-x}}\,dx$ diverges
4. $\displaystyle\int_0^\infty \frac{x}{(x^2+2)^2}\,dx$ converges to $1/4$
5. $\displaystyle\int_4^\infty e^{-y/2}\,dy$ converges to $2e^{-2}$
6. $\displaystyle\int_{-\infty}^{-1} e^{-2t}\,dt$ diverges
7. $\displaystyle\int_{2\pi}^\infty \sin(\theta)\,d\theta$ diverges
8. $\displaystyle\int_{-\infty}^\infty x^2e^{-x^3}\,dx$ diverges
9. $\displaystyle\int_1^\infty \frac{\ln x}{x}\,dx$ diverges
10. $\displaystyle\int_1^\infty \frac{\ln x}{x^2}\,dx$ converges to 1
11. $\displaystyle\int_1^\infty \frac{\ln x}{x^3}\,dx$ converges to $1/4$
12. $\displaystyle\int_2^3 \frac{1}{\sqrt{3-x}}\,dx$ converges to 2
13. $\displaystyle\int_6^8 \frac{4}{(x-6)^3}\,dx$ diverges
14. $\displaystyle\int_0^1 \frac{1}{4y-1}\,dy$ diverges

15. $\displaystyle\int_{-1}^{1} \frac{e^x}{e^x - 1}\, dx$ diverges

16. $\displaystyle\int_{0}^{1} \frac{\ln x}{\sqrt{x}}\, dx$ converges to -4

17. Find the values of p for which the integral

$$\int_2^\infty \frac{1}{x(\ln x)^p}\, dx$$

converges and evaluate the integral for those values of p. The integral converges to $\frac{1}{p-1}(\ln 2)^{1-p}$ for $p > 1$ and diverges for $p \leq 1$.

Use the Comparison Theorem to determine whether each integral is convergent or divergent.

1. $\displaystyle\int_1^\infty \frac{2 + e^{-x}}{x}\, dx$ diverges

2. $\displaystyle\int_1^\infty \frac{1}{x^2 + e^{2x}}\, dx$ converges

3. $\displaystyle\int_1^\infty \frac{x}{\sqrt{1 + x^6}}\, dx$ converges

4. $\displaystyle\int_1^\infty \frac{x^3}{x^5 + 2}\, dx$ converges