Math 333
Homework 8 Solutions
Forced Harmonic Oscillators

Note: This homework is due in class on Tuesday, April 1, 2008.

1. Consider the differential equation
$$y'' + 4y' + 20y = 3 + 2\cos(2t).$$
   (a) Find the general solution of the differential equation.
   Solution. $y(t) = k_1 e^{-2t}\sin(4t) + k_2 e^{-2t}\cos(4t) + \dfrac{1}{10}\cos(2t) + \dfrac{1}{20}\sin(2t) + \dfrac{3}{20}$
   (b) Discuss the long-term behavior of solutions of the equation.
   Solution. As $t \to \infty$, $y(t)$ approaches the steady-state solution $y_p(t) = \frac{1}{10}\cos(2t) + \frac{1}{20}\sin(2t) + \frac{3}{20}$.

2. Consider the differential equation
$$y'' + 6y' + 8y = -4\cos(3t).$$
   (a) Find the general solution of the differential equation.
   Solution. $y(t) = k_1 e^{-4t} + k_2 e^{-2t} + \dfrac{4}{325}\cos(3t) - \dfrac{72}{325}\sin(3t)$
   (b) Discuss the long-term behavior of solutions of the equation.
   Solution. As $t \to \infty$, $y(t)$ approaches the steady-state solution $y_p(t) = \frac{4}{325}\cos(3t) - \frac{72}{325}\sin(3t)$

3. Consider the differential equation
$$y'' + 2y' + y = 2\cos(2t).$$
   (a) Find the general solution of the differential equation.
   Solution. $y(t) = k_1 e^{-t} + k_2 t e^{-t} - \dfrac{6}{25}\cos(2t) + \dfrac{8}{25}\sin(2t)$
   (b) Discuss the long-term behavior of solutions of the equation.
   Solution. As $t \to \infty$, $y(t)$ approaches the steady-state solution $y_p(t) = -\frac{6}{25}\cos(2t) + \frac{8}{25}\sin(2t)$