
Practice Problems: Euler Equations

In each of the following problems, determine the general solution of the differential equation that is valid in any interval not containing the singular point.

1. $x^2y'' + 4xy' + 2y = 0$
 2. $x^2y'' - 3xy' + 4y = 0$
 3. $x^2y'' - xy' + y = 0$
 4. $x^2y'' + 3xy' + 5y = 0$
 5. $(x + 1)^2y'' + 3(x + 1)y' + 0.75y = 0$. Hint: make the substitution $t = x + 1$.
 6. $(x - 1)^2y'' + 8(x - 1)y' + 12y = 0$
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Solutions.

1. $y = c_1x^{-1} + c_2x^{-2}$
2. $y = c_1x^2 + c_2x^2 \ln |x|$
3. $y = c_1x + c_2x \ln |x|$
4. $y = c_1x^{-1} \cos(2 \ln |x|) + c_2x^{-1} \sin(2 \ln |x|)$
5. $y = c_1|x + 1|^{-1/2} + c_2|x + 1|^{-3/2}$
6. $y = c_1(x - 1)^{-3} + c_2(x - 1)^{-4}$