Math 347

Homework 7: Continuous Dynamical Systems

Due Thursday, October 18, 2007

Note: We will discuss Problems 2 and 3 in class on Tuesday. Please think about the problems over the weekend, and be prepared to discuss and present models in class on Tuesday.

1. Reconsider Problem 7 from Section 4.4. For each of the equilibrium points that you found in part (b), use the eigenvalue method to determine if the equilibrium point is stable or unstable. Use Maple to draw a phase plot with several different initial conditions around each equilibrium point to graphically illustrate stability.

2. Section 4.4 #6: The blue and fin whale are two similar species that inhabit the same areas and compete for resources. The intrinsic growth rate of each species is estimated to be 5% for the blue whale and 8% for the fin whale (per year). The environmental carrying capacity for each species is estimated at 150,000 blue whales and 400,000 fin whales. Suppose that the effect of competition between the two species is given by the parameter $\alpha = \alpha_1 = \alpha_2 = 10^{-8}$. In this problem, we will investigate the effects of harvesting on the two whale populations. Assume that a level of effort of E boat-days will result in the annual harvest of qEx_1 blue whales and qEx_2 fin whales, where x_1 represents the current number of blue whales and x_2 represents the current number of fin whales. Assume that the parameters q (which represents catchability) is equal to 10^{-5}.

   (a) Develop a continuous dynamical system model for this problem.

   (b) Under what conditions (on E) can the two species continue to coexist in the presence of harvesting?

   (c) Choose some value of E in the range that you obtained in part(b), and use Maple to draw the vector field for this problem. Also draw the phase plot for various initial conditions.

   (d) Using the value of E that you used in part (c), what are the equilibrium points for your model? Use eigenvalue methods to determine whether or not the equilibrium points are stable or unstable, and use Maple to draw phase plots around the equilibrium points to graphically illustrate stability.

   (e) Find the minimum level of harvesting effort required to reduce the fin whale population to its current level of around 70,000 whales. Assume that we started out with 150,000 blue whales and 400,000 fin whales (i.e. the carrying capacity of the two populations) before people began to harvest them.

(f) What would happen to the two populations if harvesting was allowed to continue at the level of effort identified in the part (e). Draw the vector field in this case. Note: this is the situation that led the International Whaling Commission (IWC) to call for an international ban on whaling.

3. A population of 100,000 members is subject to a disease that is seldom fatal and leaves the victim immune to future infections by the disease. Infection can only occur when a susceptible person comes in direct contact with an infectious person. The infectious period lasts approximately three weeks. Last week, there were 18 new cases of the disease reported. This week, there were 40 new cases. It is estimated that 30% of the population is immune due to previous exposure.

   (a) Develop a continuous dynamical system model for this problem.

   (b) What is the eventual number of people who will become infected?