

## Math 347

### Introductory Information: Disease Modeling

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- The SIR model of the spread of a disease considers three populations:

$S$     susceptible population  
 $I$     infected population  
 $R$     recovered population

We assume that  $S + I + R = C$ , where  $C$  is the total size of the population. The dynamic system model is given by the following differential equations, where  $r$  and  $\gamma$  are positive constants:

$$\begin{aligned}\frac{dS}{dt} &= -rSI \\ \frac{dI}{dt} &= rSI - \gamma I \\ \frac{dR}{dt} &= \gamma I\end{aligned}$$

What do each of the parameters represent? What if we modify the system of differential equations to become ( $\beta$  is a positive constant)

$$\begin{aligned}\frac{dS}{dt} &= -rSI \\ \frac{dI}{dt} &= rSI - \gamma I \\ \frac{dR}{dt} &= \gamma I - \beta R\end{aligned}$$

What new behavior is included?

- The SIQR model of the spread of a disease assumes that some of the infectives are placed in quarantine. Let  $Q$  be the size of the population in quarantine, and let  $I$  be the size of the population of infectives that are *not* in quarantine. The total population is then  $S + I + Q + R = C$ . In this model, infectives can move either to the quarantined population or to the recovered population, and people in the quarantined population move in to the recovered population. Suppose that the rate at which infectives are quarantined is proportional to  $I$ , and that the population in quarantine recovers at the same rate as the nonquarantined infectives. From this information, you can write down the system of differential equations for the SIQR model.

One situation that you might use for data is the Hong Kong Flu which occurred in the United States during the winter of 1968-69. At that time, no flu vaccine was available, so many more people were infected than would be the case today. The following table gives the weekly totals

of excess pneumonia-influenza deaths; that is, the number of such deaths in excess of the average deaths to be expected from other sources.

<b>Week</b>	<b>Flu-related deaths</b>
1	14
2	28
3	50
4	66
5	156
6	190
7	156
8	108
9	68
10	77
11	33
12	65
13	34

Feel free to incorporate this data, as well as any other epidemic-related data that you find in other sources, in your model development and discussion. If you do use this data, you should argue that you have used it appropriately.