Math 347
Discrete Dynamical Systems Homework 1

Note: These problems are due on Thursday, October 25.

1. Consider the discrete dynamical system

$$A(n+1) = 0.8A(n) - 2.$$

Find the equilibrium value (if one exists) of the system, sketch the cobweb graph, and use the cobweb to determine the stability of the equilibrium value.

2. Consider the discrete dynamical system

$$A(n+1) = 3A(n) - A^2(n) + 3.$$

Find the equilibrium values for the dynamical system, and determine the stability of the equilibrium values using cobwebs.

3. Consider the discrete dynamical system

$$A(n+1) = 3A(n) - A^2(n) - 1.$$

Find its equilibrium value a. Compute $A(n)$ for the first 5 values of n using $A(0) = a + 0.1$. Based on these computations, do you think a is a stable equilibrium value? Next, compute $A(n)$ for the first 5 values of n using $A(0) = a - 0.1$. Based on these computations, do you think a is a stable equilibrium value? You should find that a does not satisfy the definition of stable or unstable, since for some $A(0)$ values sufficiently close to a, $A(k)$ tends towards a, while for others $A(k)$ goes away from a. Such an equilibrium value is called **semistable**.

4. Consider the discrete dynamical system

$$A(n+1) = -A(n) + 4.$$

Find the equilibrium value of the system, and sketch a cobweb graph for the system using $A(0) = 6$.

5. Consider the general affine first-order dynamical system

$$A(n+1) = rA(n) + b.$$

Show that the equilibrium value for this system is

$$a = \frac{b}{1-r}$$

if $r \neq 1$, and there is no equilibrium value if $r = 1$. Next, show using the definition that the equilibrium value is stable if $|r| < 1$ and unstable if $|r| > 1$. What happens if $r = -1$? Hint: For the stability proofs, it may help to show (by induction on k) that

$$|A(k) - a| = |r|^k |A(0) - a|.$$