Math 347
Discrete Dynamical Systems Homework 4

Due Tuesday, November 6, 2007

The cobweb theorem of economics. In this series of exercises, we will develop a discrete dynamical system to model supply and demand as it relates to a product that takes one unit of time to produce. The model was originally developed to study the farming industry in which the product (a crop) is produced one year, and the farmer then plans next year's crops using information about prices this year. For example, suppose we own a farm and wish to decide how much acreage to devote to corn. If the price of corn is high after this year's harvest, then we will plant a large amount of corn next year. With a large harvest of corn next year, the price of corn will have to drop in order to create enough demand to sell the entire harvest. With the price dropping next year, people will not plant much corn the year after. With a small harvest that year, the price will rice since not much demand is needed to sell the entire crop. Notice that this is a recursive process and that the price oscillates between high and low values. But does it oscillate to equilibrium or with increasing magnitude? Or does it exhibit some other type of behavior?

To develop our model, we need to consider three quantities: the supply $S(n)$ of our product, the demand $D(n)$ for our product, and the price $P(n)$ for one unit of our product, all in year n. We will make the following set of assumptions about the relationships between supply, demand, and price.

- The supply of the product in any year depends positively on the price of the product the previous year. Assume that the relationship is linear, and that the supply equation should contain a constant (affine) term to account for such factors as increased ability to produce.
- The demand for the product in any year depends negatively on the present price of the product. Assume that the relationship is linear, and that the demand equation should contain a constant (affine) term to account for such factors as increase in population.
- Each year, the price of the product is adjusted so that the demand equals the supply.

1. Write a dynamical system of three equations to model this economic situation. Explain (in words) the meaning of any constants or parameters that you use.

2. Use your three equations to find a first order dynamical system model for $P(n+1)$ in terms of $P(n)$.

3. Find the equilibrium value for your dynamical system, and determine the conditions under which the equilibrium value is stable. Interpret these conditions in words in terms of the economic situation.

4. Find the general solution of the system.

5. Discuss the long-term trends of the solution (for various initial conditions and various relationships between the parameters). Interpret these results.

6. Suppose we have followed the price of a product for several years. We observed that when the price was 8 dollars per bushel in one year, the demand that year was 6000 bushels and the supply was 12000 bushels the next year. We also observed that when the price was 5 dollars per bushel in one year, the demand that year was 9000 bushels and the supply was 10000 bushels the next year. Determine the equilibrium price and its stability, and discuss the long-term trends of this system.