Math 347
Discrete Dynamical Systems Homework 2

Due Tuesday, October 30, 2007

1. Consider the dynamical system
$$A(n+1) = 2A(n) - 0.25A^2(n) - 0.75.$$
Show that the two equilibrium values of this system are $a = 1$ and $a = 3$, and show using derivative techniques that $a = 1$ unstable and $a = 3$ is stable. Sketch a cobweb for this system, and use it to determine the maximum interval containing $a = 3$ such that if $A(0)$ is in that interval, then $\lim_{k\to\infty} A(k) = 3$.

2. Consider the following discrete logistic population growth model
$$A(n+1) - A(n) = rA(n)(1 - \frac{A(n)}{L}),$$
where r is the unrestricted growth rate of the population and L is the environmental carrying capacity of the population. Find the equilibrium points of this system, and use derivative techniques to discuss the stability of each equilibrium point.

3. In this problem, we will consider two different harvesting strategies. Assume that we are modeling a population of deer, and that in the absence of harvesting, the deer population follows logistic growth as in the previous problem. Assume that the size of the units are chosen so that one unit equals the carrying capacity of the population ($L = 1$). Let's say that one unit is equal to 10,000 deer (so the carrying capacity for deer is 10,000). Assume that the unrestricted growth rate of the deer population is $r = 0.8$.

   (a) Consider a harvesting strategy in which hunters are allowed to kill b units of deer per season. Construct the dynamical system that models the growth of the deer population. Find the equilibrium points of the system, and use derivative and/or graphical techniques to discuss stability. What is the maximum sustainable harvest level for this population (i.e. the maximum harvest level such that the deer population does not die out).

   (b) Next, consider a harvesting strategy in which hunters are allowed to hunt a fixed proportion of the population per season. Let b represent the proportion of the population that is allowed to be hunted, and construct the dynamical system that models the growth of the deer population in this case. Find the equilibrium points of this system, and use derivative and/or graphical techniques to discuss stability. What is the maximum sustainable harvest level?

(c) Compare (in a written discussion) the two harvesting strategies.

4. As an environmental engineer, you have been asked to analyze the dumping of a pollutant in a nearby lake. You have found that the amount of pollution entering the lake from all sources is 20 pounds per year. Fortunately, sunlight gradually breaks down the pollutant into harmless by-products; each year, 10% of the pollutant is lost by this process. Currently in the lake, there are 0.2 pounds of pollutant per cubic mile of water. Safe levels have been defined by the EPA to be 0.1 pounds per cubic mile. The lake contains 1500 cubic miles of water.

   (a) Write a discrete dynamical system to model this situation.

   (b) Will the lake ever reach safe levels? Determine your answer by using an equilibrium point and stability analysis.

   (c) Now suppose that you had a method to remove more pollution so that 20% of the pollutant was removed each year. Will the lake ever reach safe levels? Justify. If so, determine how many years it will take.

   (d) Suppose that you were wrong about the initial model, and that in fact the amount of pollutant broken down by sunlight is $1.64\sqrt{P_n}$ pounds per year, where P_n is the pollution level in the n-th year. Will the lake ever reach safe levels? If so, how many years will it take?

5. Consider the dynamical system

$$A(n+1) = 3.3A(n) - 3.3A^2(n).$$

   Find a 2-cycle for the system, and determine whether it is stable or unstable.

6. Consider the dynamical system

$$A(n+1) = 3.5A(n) - 2.5A^2(n).$$

   (a) Show that $a_1 = 0.701237896$ is one point in a 4-cycle.

   (b) Show that this 4-cycle is stable (by developing a criterion similar to the one developed in class for 2-cycles).