

## Convergence of Improper Integrals

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**The p-test for improper integrals:** The improper integral

$$\int_1^{\infty} \frac{1}{x^p} dx$$

converges to  $\frac{1}{p-1}$  if  $p > 1$  and diverges if  $p \leq 1$ .

**The Comparison Theorem:** Suppose that  $f$  and  $g$  are continuous functions with  $f(x) \geq g(x) \geq 0$  for  $x \geq a$ .

(a) If  $\int_a^{\infty} f(x) dx$  is convergent, then  $\int_a^{\infty} g(x) dx$  is convergent.

(b) If  $\int_a^{\infty} g(x) dx$  is divergent, then  $\int_a^{\infty} f(x) dx$  is divergent.

A similar result holds for improper integrals with discontinuous (unbounded) integrands.

**The Absolute Comparison Theorem:** Suppose that  $\int_a^{\infty} |f(x)| dx$  converges. Then  $\int_a^{\infty} f(x) dx$  also converges.

### Examples.

1. Does

$$I = \int_1^{\infty} \frac{1}{x^5 + 1} dx$$

converge or diverge?

2. Does

$$I = \int_1^{\infty} \frac{2x + 1}{\sqrt{x} - \frac{1}{2}} dx$$

converge or diverge?

3. Does

$$I = \int_1^{\infty} \frac{1 + e^{-x}}{x} dx$$

converge or diverge?

4. Does

$$I = \int_1^{\infty} \frac{\cos^2 x}{1 + x^2} dx$$

converge or diverge?

5. Does

$$I = \int_1^{\infty} \frac{\sin x}{x^2} dx$$

converge or diverge?