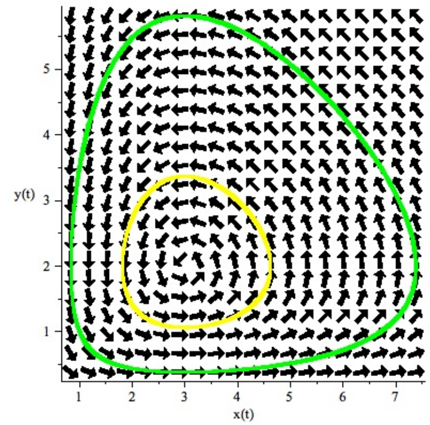
Conclusions

- Numerical and graphical simulation

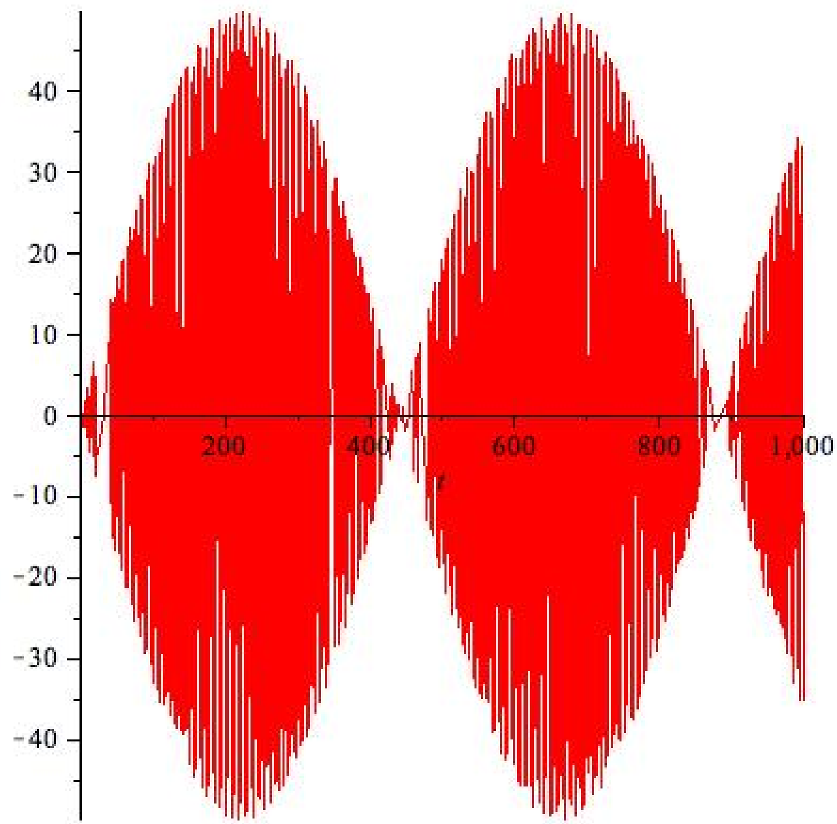


- Applications of differential equations

- Analytic methods

Numerical Analysis

- What algorithms do Maple and other computer algebra systems use to simulate differential equations?
- How can we design algorithms to maximize efficiency, stability, and accuracy?



Dynamical Systems: Applications of Differential Equations

- Predator-prey systems

$$
\begin{aligned}
\frac{dF}{dt} &= -k_F F\left(1-\frac{F}{K_F}\right)+\alpha F R\\
\frac{dR}{dt} &= k_R R\left(1-\frac{R}{K_R}\right)+-\beta F R
\end{aligned}
$$

Use the mathematical models to predict long-term behavior of ecosystems. Incorporate such factors as hunting, illness, migration, weather, competition and/or cooperation with other species, etc.

- Infectious disease models:

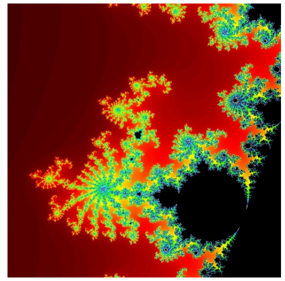
  The SIR model of the spread of a disease considers three populations:

S	susceptible population
I	infected population
R	recovered population

  The differential equation model is given by the following:

$$\begin{aligned} \frac{dS}{dt} &= -rSI \\ \frac{dI}{dt} &= rSI - \gamma I \\ \frac{dR}{dt} &= \gamma I \end{aligned}$$

- Weather prediction and chaos theory



- Analysis of the stock market

- Models of warfare

- Mathematical biology

- Models of love

Discrete dynamical systems

Use finite difference equations instead of continuous differential equations.

Differential equation:

$$\frac{dP}{dt} = kP$$

Discrete dynamical system:

$$P(n+1) = kP(n)$$

Applications:

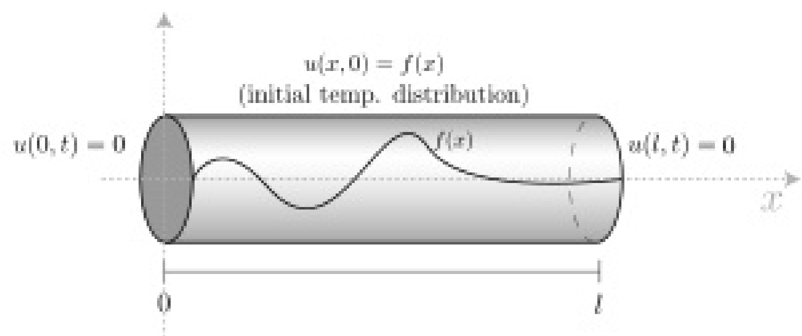
- Population modeling for species in which all births occur in spring and most deaths occur in winter. For such species, the assumption of continuous population change is not valid.
- Integer sequences (number theory)

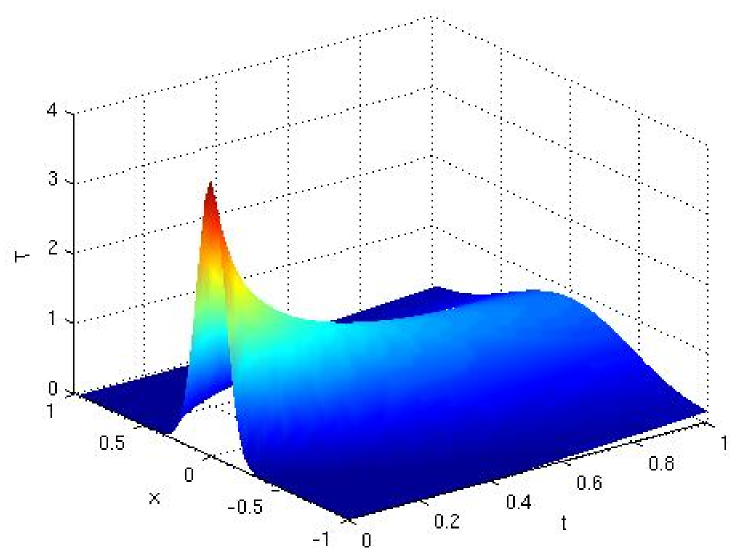
Partial Differential Equations

The heat equation:

$$u_t = ku_{xx},$$

where $u = u(t, x)$ is a function of two variables t and x





Complex Analysis, Fourier Transforms and Laplace Transforms

- The **Fourier transform** of the function $y(t)$ is

$$\int_{\infty}^{\infty} y(t)e^{-i\omega t}\,dt.$$

It measures the extent to which $y(t)$ is oscillating with frequency $\omega/2\pi$.

- The **Laplace transform** of the function $y(t)$ is

$$\int_{0}^{\infty} y(t)e^{-st}\,dt.$$

It uses integration to compare $y(t)$ to the exponential function e^{st}.

- Both transforms have important applications in differential equations, physics, optics, electrical engineering, signal processing, image processing, and probability.