Graph Coloring.

1. Find the chromatic number of each of the following graphs. Give a careful argument to show that fewer colors will not suffice.

2. A graph G is said to be *color critical* if the removal of any vertex of G decreases the chromatic number. Which graphs in Exercise 1 are color critical?

3. A mathematics department plans to offer seven graduate courses next semester: combinatorics (C), group theory (G), linear programming (L), numerical analysis (N), probability (P), statistics (S), and topology (T). The mathematics graduate students and the courses they plan to take are:

   - Adrienne: C, L, T
   - Minyu: C, G, S
   - Ariela: G, N
   - Steve: C, L
   - Brook: L, N
   - Jim: C, G
   - Dana: N, P
   - Bob: G, L
   - Carol: C, T
   - Judy: C, S, T
   - Brian: P, S
   - Brad: P, T

   How many time periods are needed for these seven courses?

4. A set of solar experiments are to be made at observatories. Each experiment begins on a given day of the year and ends on a given day. An observatory can perform only one experiment at a time. Suppose experiment A runs from Sept. 2 to Jan. 3, experiment B from Oct. 15 to March 10, experiment C from Nov. 20 to Feb. 17, experiment D from Jan. 23 to May 30, experiment E from April 4 to July 28, experiment F from April 30 to July 28, and experiment G from June 24 to Sept. 30. Find the minimum number of observatories needed to perform all of the experiments each year.

5. What is the minimum number of colors needed to color a map of the United States? For over 100 years, mathematicians have conjectured that no map, no matter how complicated, requires more than four colors. In 1976, Appel and Haken showed that every map can indeed be colored with four or fewer colors. However, due to the unusual nature of their solution, this result did not end interest in the map coloring problem. Appel and Haken solved the Four Color Problem by dividing the problem into nearly two thousand countries, according to the arrangements of countries within a map. They wrote computer programs to analyze the various colorings in each arrangement, and after 1200 hours of computer calculations, they solved the problem. Even though the solution of the Four Color Problem is a monumental achievement, many mathematicians are dissatisfied with (and some even skeptical of) the proof. Thus a new problem arose: does their exist a purely mathematical proof, unaided by computers, showing that every map can be colored with four or fewer colors?