§5.8 Chapter Summary

1. (a) Note that $f(x) < 0$ for all x in the interval $[0, 40]$. Furthermore, it is clear from the picture that $f(x) < -55$ (the area of eleven dotted rectangles). Thus, the best estimate of $\int_0^{40} f(x)\,dx$ is -65.

   (b) $\dfrac{\int_{10}^{30} f(x)\,dx}{30-10} \approx \dfrac{-37}{20} = -1.85$

2. $\int_6^{10} h(x)\,dx < h'(5) < \frac{1}{10}\big(h(10) - h(0)\big) < \int_5^5 h(x)\,dx < \frac{1}{10}\int_0^{10} h(x)\,dx < \int_0^{10} h(x)\,dx < \int_0^5 h(x)\,dx$ (i.e., (vi) < (i) < (iii) < (vii) < (ii) < (iv) < (v))

3. (a) $y = -2$ is one possibility.

   (b) $y = 8 - 20x$ is one possibility.

4. Let $f(x) = \sin(e^x)$. Then, $0.4 < \sin(e) \le f(x) \le 1$ if $0 \le x \le 1$. Therefore, $0.4 \le \displaystyle\int_0^1 f(x)\,dx \le 1$.

5. If $0 \le x \le \pi$, $\cos 1 \le \cos(\sin x) \le 1$. Therefore,

$$\frac{\pi}{2} < \cos 1 \cdot (\pi - 0) \le \int_0^{\pi} \cos(\sin x)\,dx \le 1 \cdot (\pi - 0) = \pi.$$

6. (a) If $0 \le x \le \pi$, then $0 \le \sin x \le 1 \implies 1 \le e^{\sin x} \le e$. Thus, $\pi \le \int_0^{\pi} e^{\sin x}\,dx \le \pi e$.
   If $\pi \le x \le 2\pi$, $-1 \le \sin x \le 0 \implies 1/e \le e^{\sin x} \le 1$. Thus, $\pi/e \le \int_0^{\pi} e^{\sin x}\,dx \le \pi$.
   Since $\int_0^{2\pi} e^{\sin x}\,dx = \int_0^{\pi} e^{\sin x}\,dx + \int_{\pi}^{2\pi} e^{\sin x}\,dx$, the inequalities above lead to $\pi(1 + 1/e) \le \int_0^{2\pi} e^{\sin x}\,dx \le \pi(1+e)$.

   (b) Since the integrand is periodic with period 2π, $25\pi(1 + 1/e) \le \displaystyle\int_0^{50\pi} e^{\sin x}\,dx \le 25\pi(1+e)$.

7. (a) Since the graph of the arctangent function is concave down, the secant line through the points $(0, \arctan 0) = (0, 0)$ and $(1, \arctan 1) = (1, \pi/4)$ lies below the curve $y = \arctan x$.

   (b) Let $f(x) = \arctan x$. Then $f(0) = 0$, $f'(x) = 1/(1 + x^2)$, and so $f'(0) = 1$. Therefore, the equation of the tangent line is $y = x$.

   (c) Let g be secant line from part (a) $[g(x) = \pi x/4]$ and $h(x) = x$ be the tangent line from part (b). Then $g(x) \le \arctan x \le h(x)$ if $0 \le x \le 1$. Therefore,
   $\dfrac{\pi}{8} = \displaystyle\int_0^1 g(x)\,dx \le \int_0^1 \arctan x\,dx \le \int_0^1 h(x)\,dx = \frac{1}{2}.$

8. (a) Since $0 \le x^k f(x) \le f(x)$ if $0 \le x \le 1$ and $k \ge 0$ is an integer, $\int_0^1 x^k f(x)\,dx \le \int_0^1 f(x)\,dx$.

   (b) No. For example, let $f(x) = -1$. Then $-1 = \int_0^1 f(x)\,dx < \int_0^1 x f(x)\,dx = -1/2$.

9. (a) $\displaystyle\int_0^{\pi/2} \sin^2 x\,dx + \int_0^{\pi/2} \cos^2 x\,dx = \int_0^{\pi/2} \left(\sin^2 x + \cos^2 x\right)\,dx = \int_0^{\pi/2} dx = \frac{\pi}{2}$

   (b) $\displaystyle\int_0^{\pi/2} \cos^2 x\,dx = \int_0^{\pi/2} \sin^2(x - \pi/2)\,dx = \int_{-\pi/2}^{0} \sin^2 x\,dx = \int_0^{\pi/2} \sin^2 x\,dx$

   (c) $\displaystyle\int_0^{\pi/2} \sin^2 x\,dx + \int_0^{\pi/2} \cos^2 x\,dx = 2\int_0^{\pi/2} \sin^2 x\,dx = \frac{\pi}{2}$, so $\displaystyle\int_0^{\pi/2} \sin^2 x\,dx = \frac{\pi}{4}$.

10. (a) $\sqrt{1 + \cos(2x)} = \sqrt{2\cos^2 x} = \sqrt{2}\cos x$ if $0 \le x \le \pi/2$.

    (b) No, since $\int_{\pi/2}^{\pi} \sqrt{1 + \cos(2x)}\,dx = -\sqrt{2}\int_{\pi/2}^{\pi} \cos x\,dx$.

11. The bounds on f imply that $-4 \le \int_1^3 f(x)\,dx \le 10$. Thus, $-2 \le \dfrac{\int_1^3 f(x)\,dx}{2} \le 5$.

12. Let $V(c) = \int_a^b \left(f(x) - c\right)^2 dx = \int_a^b \left(f(x)\right)^2 dx - 2c\int_a^b f(x)\,dx + c^2 \int_a^b dx$. Now $V'(c) = -2\int_a^b f(x)\,dx + 2c(b-a)$ is zero if $c = \left(\int_a^b f(x)\,dx\right)/(b-a)$ (i.e., if c is the average value of f on the interval $[a, b]$). Since $V''(c) = 2(b-a) > 0$, this value of c corresponds to the minimum value of V.

13. (a) No. Let $f(x) = 0$ and $g(x) = x$. Then $\int_{-1}^1 f(x)\,dx = \int_{-1}^1 g(x)\,dx = 0$ but $f(x) \ge g(x)$ if $-1 \le x \le 0$.

    (b) Yes. If $f(x) > g(x)$ for *every* x such that $a \le x \le b$, then $\int_a^b f(x)\,dx > \int_a^b g(x)\,dx$ would be true — a contradiction.

14. $G(x) = \int_b^x f(t)\,dt = \int_a^x f(t)\,dt - \int_a^b f(t)\,dt = F(x) + C$ where $C = -\int_a^b f(t)\,dt$.

15. $\sin^2 x = 1 - \cos^2 x \implies \sin^2 x + C_1 = \left(1 - \cos^2 x\right) + C_1 = -\cos^2 x + (1 + C_1) = -\cos^2 x + C_2$

16. $\frac{1}{2}\sin(2x) = \frac{1}{2}\cdot 2\cos x \sin x = \cos x \sin x$

17. $\frac{1}{2}\cos x \sin x = \frac{2\cos x \sin x}{4} = \frac{1}{4}\sin(2x)$

18. (a)
$$\int \sin^2 x\,dx = \int \frac{1}{2}(1 - \cos(2x))\,dx = \frac{1}{2}\left(\int 1\,dx - \int \cos(2x)\,dx\right)$$
$$= \frac{1}{2}\left(x - \frac{1}{2}\sin(2x)\right) + C = \frac{x}{2} - \frac{1}{4}\sin(2x) + C$$

    (b) $\sqrt{1 - \cos(2x)} \ne \sqrt{2}\sin x$ if $\pi < x < 2\pi$.

19. $\left(x\ln x - x + C\right)' = \ln x + \frac{x}{x} - 1 = \ln x$

20. $\left(x\arctan x - \frac{1}{2}\ln\left(1 + x^2\right) + C\right)' = \arctan x + \frac{x}{1+x^2} - \frac{1}{2}\cdot\frac{2x}{1+x^2} = \arctan x$

21. $\left(\frac{1}{a}\arctan\left(\frac{x}{a}\right) + C\right)' = \frac{1}{a}\cdot\frac{1/a}{1 + (x/a)^2} = \frac{1}{a^2 + x^2}$

22. $\left(\ln|\sec x| + C\right)' = \frac{\sec x \tan x}{\sec x} = \tan x$

23. $\left(\ln|\sec x + \tan x| + C\right)' = \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x} = \frac{\sec x\,(\tan x + \sec x)}{\sec x + \tan x} = \sec x$

24. $\left(\frac{1}{2}\ln\left|\frac{1+x}{1-x}\right| + C\right)' = \frac{1}{2}\cdot\frac{1-x}{1+x}\cdot\frac{2}{(1-x)^2} = \frac{1}{1-x^2}$

25. $\left(x\arcsin x + \sqrt{1-x^2} + C\right)' = \arcsin x + \frac{x}{\sqrt{1-x^2}} + \frac{-2x}{2\sqrt{1-x^2}} = \arcsin x$

26. $\int \left(3x^5 + 4x^{-2}\right) dx = \frac{1}{2}x^6 - 4/x + C$

27. $\int \frac{dx}{3x} = \frac{1}{3}\ln|x| + C$

28. $\int \frac{dx}{4\sqrt{1-x^2}} = \frac{1}{4}\arcsin x + C$

29. $\int \frac{3}{x^2+1}\,dx = 3\arctan x + C$

30. $\int 3e^{4x}\,dx = \frac{3}{4}e^{4x} + C$

31. $\int \left(2\sin(3x) - 4\cos(5x)\right)dx = -\frac{2}{3}\cos(3x) - \frac{4}{5}\sin(5x) + C$

32. $\int 4\sec^2(3x)\,dx = \frac{4}{3}\tan(3x) + C$

33. $\int \left(1+\sqrt{x}\right)^2 dx = \int (1 + 2\sqrt{x} + x)\,dx = x + \frac{4}{3}x^{3/2} + \frac{1}{2}x^2 + C$

34. $\int (x+1)^2\sqrt[3]{x}\,dx = \int \left(x^{7/3} + 2x^{4/3} + x^{1/3}\right) dx = \frac{3}{10}x^{10/3} + \frac{6}{7}x^{7/3} + \frac{3}{4}x^{4/3} + C$

35. $\int \frac{(3-x)^2}{x}\,dx = \int \frac{9-6x+x^2}{x}\,dx = 9\ln|x| - 6x + \frac{1}{2}x^2 + C$

36. $\int \frac{(1-x)^3}{\sqrt{x}}\,dx = \int \left(x^{-1/2} - 3x^{1/2} + 3x^{3/2} - x^{5/2}\right) dx = 2x^{1/2} - 2x^{3/2} + \frac{6}{5}x^{5/2} - \frac{2}{7}x^{7/2} + C$

37. $\int e^x(1-e^x)\,dx = \int \left(e^x - e^{2x}\right) = e^x - \frac{1}{2}e^{2x} + C$

38. Let $u = (x-1)$. Then,

$$\int \frac{dx}{\sqrt{2x-x^2}} = \int \frac{dx}{\sqrt{1-(x-1)^2}} = \int \frac{du}{\sqrt{1-u^2}} = \arcsin u = \arcsin(x-1)$$

39. Let $u = x^2$. Then $du = 2x\,dx$ so $\int x\sin(x^2)\,dx = \frac{1}{2}\int \sin u\,du = -\frac{1}{2}\cos u + C = -\frac{1}{2}\cos(x^2) + C.$

40. $\int \frac{dx}{2x+1} = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|2x+1| + C$

41. Let $u = x^4$. Then $du = 4x^3\,dx$ so $\int x^3e^{x^4}\,dx = \frac{1}{4}\int e^u\,du = \frac{1}{4}e^u + C = \frac{1}{4}e^{x^4} + C.$

42. Using the subsitution $u = \arctan x$, $\int \frac{\arctan x}{1+x^2}\,dx = \int u\,du = \frac{1}{2}u^2 = \frac{1}{2}(\arctan x)^2 + C.$

43. Using the substitution $u = \ln x$, $\int \frac{(\ln x)^3}{x}\,dx = \int u^3\,du = \frac{1}{4}u^4 = \frac{1}{4}(\ln x)^4 + C.$

44. Using the substitution $u = 3x$, $\int \frac{dx}{1+9x^2} = \frac{1}{3}\int \frac{du}{1+u^2} = \frac{1}{3}\arctan u = \frac{1}{3}\arctan(3x) + C.$

45. Let $u = \ln(\cos x)$. Then $du = -\frac{\sin x}{\cos x}\,dx = -\tan x\,dx$. Therefore, $\int \ln(\cos x)\tan x\,dx = -\int u\,du$ $= -\frac{1}{2}u^2 + C = -\frac{1}{2}\left(\ln|\cos x|\right)^2 + C.$

46. Let $u = x^2$. Then $du = 2x\,dx$. Therefore, $\int_0^1 x\sqrt{1-x^4}\,dx = \frac{1}{2}\int_0^1 \sqrt{1-u^2}\,du = \frac{1}{2}\cdot\frac{\pi}{4} = \frac{\pi}{8}$.

[NOTE: $\int_0^1 \sqrt{1-u^2}\,du$ is the area of a quarter-circle with radius 1.]

47. Let $u = 1 - x$. Then, $\int_0^1 x^n(1-x)^m\,dx = \int_1^0 (1-u)^n u^m\,(-du) = \int_0^1 u^m(1-u)^n\,du$.

48. The "proof" is flawed because $u = x^{-1}$ is not defined for $x = 0$.

49. $\sqrt{1-\sin^2 x} = -\cos x$ if $\pi/2 \le x \le \pi$. The "proof" incorrectly replaces $\sqrt{1-\sin^2 x}$ by $\cos x$ for all x in the interval $[0, \pi]$.

50. (a) $\int_0^5 r(t)\,dt$

(b) $\int_0^5 r(t)\,dt \approx 32e^{0.05} + 32e^{0.1} + 32e^{0.15} + 32e^{0.2} + 32e^{0.25} \approx 186.36$

(c) Each term approximates the amount of oil consumed during the previous year.

(d) $\int_0^5 32e^{0.05t}\,dt = \frac{32}{0.05}e^{0.05t}\Big]_0^5 = 640\left(e^{0.25}-1\right) \approx 181.78$

51. According to the Fundamental Theorem of Calculus, the change in the coffee's temperature is

$$\Delta T = \int_0^5 T'(t)\,dt = \int_0^5 \left(-3.5e^{-0.05t}\right)dt = \frac{3.5}{0.05}e^{-0.05t}\Big|_0^5 = 70\left(e^{-0.25}-1\right) \approx -15.5\,^\circ\text{C}.$$

Therefore, after 5 minutes, the temperature of the coffee is approximately 74.5 °C.

52. left: $\int_0^5 \sqrt[3]{2x}\,dx \approx \frac{5}{10}\sum_{j=0}^{9}\sqrt[3]{2\cdot j\cdot\frac{5}{10}} = \frac{1}{2}\sum_{j=0}^{9}\sqrt[3]{j}$

right: $\int_0^5 \sqrt[3]{2x}\,dx \approx \frac{5}{10}\sum_{j=1}^{10}\sqrt[3]{2\cdot j\cdot\frac{5}{10}} = \frac{1}{2}\sum_{j=1}^{10}\sqrt[3]{j}$

midpoint: $\int_0^5 \sqrt[3]{2x}\,dx \approx \frac{5}{10}\sum_{j=0}^{9}\sqrt[3]{2\cdot(j+0.5)\cdot\frac{5}{10}} = \frac{1}{2}\sum_{j=0}^{9}\sqrt[3]{j+0.5}$

53. left: $\int_0^5 \sqrt{3x}\,dx \approx \frac{5}{N}\sum_{k=1}^{N}\sqrt{3\cdot(k-1)\cdot\frac{5}{N}} = \frac{5}{N}\sum_{j=0}^{N-1}\sqrt{3\cdot j\cdot\frac{5}{N}}$

right: $\int_0^5 \sqrt{3x}\,dx \approx \frac{5}{N}\sum_{k=1}^{N}\sqrt{3\cdot k\cdot\frac{5}{N}} = \frac{5}{N}\sum_{j=0}^{N-1}\sqrt{3\cdot(j+1)\cdot\frac{5}{N}}$

midpoint: $\int_0^5 \sqrt{3x}\,dx \approx \frac{5}{N}\sum_{k=1}^{N}\sqrt{3\cdot(k-0.5)\cdot\frac{5}{N}} = \frac{5}{N}\sum_{j=0}^{N-1}\sqrt{3\cdot(j+0.5)\cdot\frac{5}{N}}$

54. (a) $\sum_{k=1}^{4}\frac{5}{k(k+1)}(2.3+(k-1)\cdot 0.5)^2$

(b) No. The endpoints of the subintervals are $x_0 = 0, x_1 = 5/2, x_2 = 10/3, x_3 = 15/4$, and $x_4 = 4$. The sampling points are $c_1 = 2.3$, $c_2 = 2.8$, $c_3 = 3.3$, and $c_4 = 3.8$. However, since c_2 and c_3 lie in the same subinterval, the sum is not a Riemann sum.

55. $\int_1^3 g(u)\,du = G(3)$

56. $\int_0^1 g(x)\,dx = -\int_1^0 g(x)\,dx = -G(0)$

57. $\int_{-2}^2 g(t)\,dt = \int_{-2}^1 g(t)\,dt + \int_1^2 g(t)\,dt = -\int_1^{-2} g(t)\,dt + \int_1^2 g(t)\,dt$
$= -G(-2) + G(2) = G(2) - G(-2)$

58. $\int_2^4 g(t)\,dt = \int_1^4 g(t)\,dt - \int_1^2 g(t)\,dt = G(4) - G(2)$

59. (a) If f is an *even* function, $f(x) = f(-x)$ so the signed area of the region enclosed by the graph of f between $x = -3$ and $x = 0$ is the same as the signed area of the region enclosed by the graph of f between $x = 0$ and $x = 3$. Thus,
$\int_{-3}^3 f(x)\,dx = \int_{-3}^0 f(x)\,dx + \int_0^3 f(x)\,dx = 2\int_0^3 f(x)\,dx = 2 \cdot -1 = -2.$

(b) If f is an *odd* function, $f(x) = -f(-x)$ so the signed area of the region enclosed by the graph of f between $x = -3$ and $x = 0$ has the same magnitude but the opposite sign as the signed area of the region enclosed by the graph of f between $x = 0$ and $x = 3$. Thus,
$\int_{-3}^3 f(x)\,dx = \int_{-3}^0 f(x)\,dx + \int_0^3 f(x)\,dx = 0.$

60. $0 \le x \sin x \le x$ if $0 \le x \le \pi$, so $0 \le \int_0^\pi x \sin x\,dx \le \int_0^\pi x\,dx = \pi^2/2$

61. Note that $h(1) = \int_0^1 g(t)\,dt$, that $h(4) = \int_0^4 g(t)\,dt$, that $\int_1^1 g(x)\,dx = 0$, and that $\int_5^3 g(t)\,dt = -\int_3^5 g(t)\,dt$. Finally, using the signed area interpretation of the definite integral, it is clear from the graph that $\int_5^3 g(t)\,dt < \int_0^2 g(u)\,du < -1 < h(1) < \int_1^1 g(x)\,dx < h(4) < 3 < 5$.

62. $h'(1) = g(1) = -1$

63. No — Since $h(x) = \int_0^x g(t)\,dt$, $h'(x) = g(x)$ and $h''(x) = g'(x)$. Therefore, $h''(3) = g'(3) > 0$ since g is increasing at $x = 3$.

64. Since $h(0) = 0$, 0 is a root of h. Now g is negative on the interval $(0, 2)$ and positive on the interval $(2, 5]$, h is decreasing on the first interval and increasing on the second. Therefore, since $h(2) < 0$ and $h(5) > 0$, h must have a root in the interval $[2, 5]$. This implies that h has two roots in the interval $[0, 5]$.

65. $\int_4^6 \big(2T(z) + 3\big)\,dz = 2\int_4^6 T(z)\,dz + 3\int_4^6 dz = 2 \cdot 15 + 3 \cdot (6 - 4) = 36$

66. $68 = \int_1^5 \big(3f(x) + 2\big)\,dx = 3\int_1^5 f(x)\,dx + 2\int_1^5 dx = 3\int_1^5 f(x)\,dx + 2 \cdot 4$. It follows that $3\int_1^5 f(z)\,dz = 60$, so $\int_1^5 f(z)\,dz = 20$.

67. $\int_1^{-3} h(w)\,dw = -\int_{-3}^1 h(w)\,dw = -(-2) = 2$

68. The average value of h over the interval $[-1, 1]$ is $\dfrac{\int_{-1}^1 h(u)\,du}{1-(-1)} = \dfrac{4}{2} = 2$.

69. $\int_{-3}^{-1} h(z)\,dz = \int_{-3}^{1} h(t)\,dt - \int_{-1}^{1} h(u)\,du = -2 - 4 = -6$

70. No. If h were an odd function, $\int_{-1}^{1} h(u)\,du = 0$ would be true.

71. (a) $H(-3) = -\int_{-3}^{-2} xe^x\,dx > 0$

[NOTE: Since $-3 \le x \le -2 \implies xe^x < 0$, $\int_{-3}^{-2} xe^x\,dx < 0$.]

(b) $H(-2) = \int_{-2}^{-2} xe^x\,dx = 0$

(c) $H(0) = \int_{-2}^{0} xe^x\,dx < 0$

[NOTE: $xe^x < 0$ for all $x < 0$.]

(d) $H(2) = \int_{-2}^{2} xe^x\,dx > 0$

[NOTE: A quick look at the graph of $h(x) = xe^x$ over the interval $[-2, 2]$ makes it clear that $\left|\int_{-2}^{0} xe^x\,dx\right| < \int_{0}^{2} xe^x\,dx$. Therefore, $\int_{-2}^{2} xe^x\,dx = \int_{-2}^{0} xe^x\,dx + \int_{0}^{2} xe^x\,dx > 0$.]

72. The minimum value of $h(x) = xe^x$ over the interval $[-2, 1]$ is $-e^{-1}$. Therefore, $H(1) = \int_{-2}^{1} h(x)\,dx \ge \int_{-2}^{1} (-e^{-1})\,dx = -3e^{-1}$.

73. $H'(w) = we^w$, so $H'(1) = e$.

74. $H''(w) = \left(we^w\right)' = (1+w)e^w$. Since $H''(w) > 0$ on the interval $(-1, \infty)$, H is concave up there.

75. $\left(\frac{3}{n}\right)\sum_{k=1}^{n}\left(1+\frac{3k}{n}\right)^2$ is a right Riemann sum approximation to the integral $\int_1^4 x^2\,dx$. Therefore,

$$\lim_{n\to\infty}\left(\frac{3}{n}\right)\sum_{k=1}^{n}\left(1+\frac{3k}{n}\right)^2 = \int_1^4 x^2\,dx = \left.\frac{x^3}{3}\right]_1^4 = 21.$$

76. Yes. Let $x_0 = -1$, $x_1 = -2/5$, $x_2 = 1/5$, $x_3 = 4/5$, $x_4 = 7/5$, $x_5 = 2$, $t_1 = -1$, $t_2 = 0$, $t_3 = 1/2$, $t_4 = 1$, $t_5 = 2$, and $f(x) = x$. Then, $x_{j-1} \le t_j \le x_j$ and $x_j - x_{j-1} = 3/5$ for each $j = 1 \ldots 5$. Thus, $\left(\frac{3}{5}\right)\cdot\left(-1+0+\frac{1}{2}+1+2\right) = \frac{3}{5}\sum_{j=1}^{5} f(t_j)$ is a Riemann sum approximation to $\int_{-1}^{2} x\,dx$.

77. Since $G(x) = \int_x^2 \cos\left(\pi t^2/4\right)dt = -\int_2^x \cos\left(\pi t^2/4\right)dt$, $G'(x) = -\cos(\pi x^2/4)$. Therefore, $G'(2) = -\cos(\pi) = 1$.

78. Let $u = u(x) = \ln x$. Then $du = x^{-1}\,dx$, $u(1) = 0$, and $u(e) = 1$, so $42 = \int_1^e \frac{g(\ln x)}{x}\,dx = \int_0^1 g(u)\,du$. Therefore, $\frac{1}{2}\int_0^1 g(u)\,du = 21$ (i.e., $K = 1/2$, $a = 0$, $b = 1$).

79. Yes — it is L_7, the left Riemann sum approximation of the integral $\int_3^5 \sqrt{x}\,dx$ computed using $n = 7$ equal subintervals (i.e., $\Delta x = (5-3)/7 = 2/7$):

$$\frac{2}{7}\sum_{k=0}^{6}\sqrt{3+2k/7} = \frac{2}{7}\left(\sqrt{3}+\sqrt{3+2/7}+\sqrt{3+4/7}+\sqrt{3+6/7}\right.$$
$$\left.+\sqrt{3+8/7}+\sqrt{3+10/7}+\sqrt{3+12/7}\right).$$

80. $\lim_{n\to\infty}\frac{5}{n}\sum_{k=1}^{n}\sqrt{4+5k/n} = \int_4^9 \sqrt{w}\,dw = \int_0^5 \sqrt{4+x}\,dx = \frac{38}{3}$. (The sum is a right Riemann sum approximation.)

81. $\int_4^1 \big(2h(z)-5\big)\,dz = -\int_1^4 \big(2h(z)-5\big)\,dz = -2\int_1^4 h(z)\,dz + 5\int_1^4 dz = -34+15 = -19$.

82. $\int_{-2}^{3} \big(f(x)+1\big)\,dx = 0 \implies \int_{-2}^{3} f(x)\,dx = -5$. Since $\int_{-2}^{3} x\,dx = 5/2$,
$\int_{-2}^{3} \big(f(x)-x\big)\,dx = -5-5/2 = -15/2$.

83. By examining a graph of $\sin(t^2/2)$ over the interval $[0, 3]$ and using the signed area interpretation of the definite integral, one can see that $-0.5 < S(0) < S(1) < 0.5 < S(3) < S(2)$.

84. No — since $\sin(4^2/2) = \sin 8 \approx 0.98936 > 0$, $S(z)$ is *increasing* at $z = 2$.

85. $S'(z) = \sin(z^2/2)$ so $S''(z) = z\cos(z^2/2)$. Therefore, $S''(3) = 3\cos(9/2) \approx -0.632 < 0$. This implies that $S(z)$ is concave down at $z = 3$.

86. $S''(z) = z\cos(z^2/2)$ so $S''(4) = 4\cos(8) \approx -0.58200$.

87. Using the signed area interpretation of the integral and a graph of the integrand, it is clear that $S(z) = 0$ for only one value of z in the interval $[0, 5]$ (i.e., at $z = 1$).

88. S attains its minimum value over the interval $[0, 5]$ at $z = 0$. (On a finite interval, S attains its extreme values at an endpoint of the interval or at a root of $S'(z) = \sin(z^2/2)$ that lies within the interval.)

89. The maximum value of S over the interval must occur at a stationary point of S or at an endpoint of the interval. Now, $S'(z) = \sin(z^2/2)$ changes sign from positive to negative only at $z = \sqrt{2\pi}$ and at $z = \sqrt{6\pi}$, so these are the only points in the interior of the interval $[0, 5]$ at which the maximum value could occur. Using the signed area interpretation of the definite integral, it is clear from a graph of the integrand that $S(0) < 0 < S(5) < S(\sqrt{6\pi}) < S(\sqrt{2\pi})$. Thus, S attains its maximum value over the interval $[0, 5]$ at $z = \sqrt{2\pi}$.

90. The average value of $\sin(t^2/2)$ over the interval $[4, 7]$ is $\frac{\int_4^7 \sin(t^2/2)\,dt}{7-4} = \frac{1}{3}\big(S(7)-S(4)\big) \approx -0.04488$.

91. $\int_2^1 f(t)\,dt = -\int_1^2 f(t)\,dt = -3$

92. $\int_2^2 f(t)\,dt = 0$

93. insufficient information is given