Math 333
Lab 2: The Tacoma Narrows Bridge

Resources: You should use Section 4.5 in your textbook as a primary reference for this lab. The article *Large-amplitude Periodic Oscillations in Suspension Bridges: Some New Connections with Non-linear Analysis*, by A.C. Lazer and P.J. McKenna, in *SIAM Review*, Vol. 32, No. 4, 1990, pp. 537–578, is posted on the course webpage (on the Labs page), and you are welcome to use it as a secondary reference. There are, of course, numerous other references on the dynamics of suspension bridges in general, and on the Tacoma Narrows Bridge in particular. You should feel free to use such additional resources as well; just make sure that you properly cite any references that you use.

Your submission for this lab should be a written report. The report should be typed (preferably in Word, LaTeX, Maple, OpenOffice, or a similar type-setting program), though you may hand-write mathematical expressions if you prefer. Please let me know if you need help finding or using a mathematical type-setting program.

I recommend organizing your report into clearly defined sections using headings and sub-headings such as *Introduction*, *Model with no wind*, *Model with wind*, *Conclusions*, *References*, etc. Provide labels and captions for any graphs and/or figures that you include in the paper, and make sure that you clearly describe and interpret your graphs in words in the main text of the paper. Your paper should contain a *Conclusions* section in which you succinctly present the main results of your work. No new results should be included in the *Conclusions* section.

The text of your report should address the items described below.

- Explain why the collapse of the Tacoma Narrows Bridge was likely *not* due to resonance.
- Present and discuss the second-order differential equation developed by Lazer and McKenna to model a suspension bridge on a calm day (no wind). Describe the meaning of each term in the differential equation. In particular, include a discussion of the meaning of numerical parameters in the differential equation.If possible, discuss a range of reasonable/realistic values for each numerical parameter. For the numerical parameters for which you do not know a range of realistic values, discuss any ideas that you might have for estimating the numbers. How does the model relate to the mass-spring-rubber band system that you studied previously?
- Convert the second-order differential equation that models the bridge on a calm day to a system of first-order differential equations.

- Simulate the system for various initial conditions. You can use the numerical parameters $a = 17$, $b = 13$, $\alpha = 0.01$, and $g = 10$ studied by Lazer and McKenna, though you should feel free to experiment with other values for these parameters as well. Discuss the behavior of the system for various initial conditions $y(0)$ and $v(0)$. You should include graphs of $y(t)$ vs. t and $v(t)$ vs. t to supplement your discussion. See the Maple file for simulation of forced harmonic oscillators posted on the Course Schedule page and on the P: drive (P:/Class/Math/Paquin/DiffEq/ForcedOscillator.mw) for the Maple syntax for graphical simulation of systems.

- Present and discuss the second-order differential equation that incorporates the effect of wind into the model. Describe the meaning of each term in the differential equation. Again, when possible, discuss a range of reasonable/realistic values for each numerical parameter.

- Convert the second-order differential equation that incorporates the effect of wind to a system of first-order differential equations.

- Simulate the system for various initial conditions, and for various values of λ and μ. You can use the numerical parameters $a = 17$, $b = 13$, $\alpha = 0.01$, and $g = 10$, as before. Discuss the behavior of the bridge as λ increases (for fixed μ) for various initial conditions. Include graphs of $y(t)$ vs. t and $v(t)$ vs. t to supplement your discussion. Find values of λ and μ and initial conditions $y(0)$ and $v(0)$ for which solutions oscillate with small amplitude (e.g. Figure 4.36), and values for which solutions oscillate with large amplitude (e.g. Figure 4.37). Include representative graphs for each situation. See the Maple file for simulation of forced harmonic oscillators posted on the Course Schedule page and on the P: drive (P:/Class/Math/Paquin/DiffEq/ForcedOscillator.mw) for the Maple syntax for graphical simulation of systems.

- Discuss your results in terms of implications for the behavior of the bridge.

- Discuss the limitations of the models that you have presented, and describe any ideas that you might have to make the models more accurate or realistic. For example, what kinds of simplifying assumptions have been made in constructing the differential equations?

- Finally, feel free to include a discussion of any of the more advanced models or research that you may have read in the Lazer/McKenna paper (or elsewhere).