B/D
Section 3.4
Complex Roots.
HW Problems for
2/19/08. See
posted assignment
for problem
numbers.

PROBLEMS

In each of Problems 1 through 6 use Euler's formula to write the given expression in the form $a + ib$.

1. $\exp(1 + 2i)$
2. $\exp(2 - 3i)$
3. $e^{i\pi}$
4. $e^{2-(\pi/2)i}$
5. 2^{1-i}
6. π^{-1+2i}

In each of Problems 7 through 16 find the general solution of the given differential equation.

7. $y'' - 2y' + 2y = 0$
8. $y'' - 2y' + 6y = 0$
9. $y'' + 2y' - 8y = 0$
10. $y'' + 2y' + 2y = 0$
11. $y'' + 6y' + 13y = 0$
12. $4y'' + 9y = 0$
13. $y'' + 2y' + 1.25y = 0$
14. $9y'' + 9y' - 4y = 0$
15. $y'' + y' + 1.25y = 0$
16. $y'' + 4y' + 6.25y = 0$

In each of Problems 17 through 22 find the solution of the given initial value problem. Sketch the graph of the solution and describe its behavior for increasing t.

17. $y'' + 4y = 0, \quad y(0) = 0, \quad y'(0) = 1$
18. $y'' + 4y' + 5y = 0, \quad y(0) = 1, \quad y'(0) = 0$
19. $y'' - 2y' + 5y = 0, \quad y(\pi/2) = 0, \quad y'(\pi/2) = 2$
20. $y'' + y = 0, \quad y(\pi/3) = 2, \quad y'(\pi/3) = -4$
21. $y'' + y' + 1.25y = 0, \quad y(0) = 3, \quad y'(0) = 1$
22. $y'' + 2y' + 2y = 0, \quad y(\pi/4) = 2, \quad y'(\pi/4) = -2$

23. Consider the initial value problem

$$3u'' - u' + 2u = 0, \quad u(0) = 2, \quad u'(0) = 0.$$

(a) Find the solution $u(t)$ of this problem.
(b) Find the first time at which $|u(t)| = 10$.

24. Consider the initial value problem

$$5u'' + 2u' + 7u = 0, \quad u(0) = 2, \quad u'(0) = 1.$$

(a) Find the solution $u(t)$ of this problem.
(b) Find the smallest T such that $|u(t)| \le 0.1$ for all $t > T$.

25. Consider the initial value problem

$$y'' + 2y' + 6y = 0, \quad y(0) = 2, \quad y'(0) = \alpha \ge 0.$$

(a) Find the solution $y(t)$ of this problem.
(b) Find α so that $y = 0$ when $t = 1$.
(c) Find, as a function of α, the smallest positive value of t for which $y = 0$.
(d) Determine the limit of the expression found in part (c) as $\alpha \to \infty$.

26. Consider the initial value problem

$$y'' + 2ay' + (a^2 + 1)y = 0, \quad y(0) = 1, \quad y'(0) = 0.$$

(a) Find the solution $y(t)$ of this problem.
(b) For $a = 1$ find the smallest T such that $|y(t)| < 0.1$ for $t > T$.
(c) Repeat part (b) for $a = 1/4, 1/2$, and 2.
(d) Using the results of parts (b) and (c), plot T versus a and describe the relation between T and a.