Section 3.2, page 151

1. $-\frac{7}{2}e^{t/2}$
2. 1
3. e^{-4t}
4. x^2e^x
5. $-e^{2t}$
6. 0
7. $0 < t < \infty$
8. $-\infty < t < 1$
9. $0 < t < 4$
10. $0 < t < \infty$
11. $0 < x < 3$
12. $2 < x < 3\pi/2$
14. The equation is nonlinear.
15. The equation is nonhomogeneous.
16. No
17. $3te^{2t} + ce^{2t}$
18. $te^t + ct$
19. $5W(f,g)$
20. $-4(t\cos t - \sin t)$
21. $y_1(t) = \frac{1}{3}e^{-2t} + \frac{2}{3}e^t, \quad y_2(t) = -\frac{1}{3}e^{-2t} + \frac{1}{3}e^t$
22. $y_1(t) = -\frac{1}{2}e^{-3(t-1)} + \frac{3}{2}e^{-(t-1)}, \quad y_2(t) = -\frac{1}{2}e^{-3(t-1)} + \frac{1}{2}e^{-(t-1)}$
23. Yes
24. Yes
25. Yes
26. Yes
27. (b) Yes
    (c) $[y_1(t), y_3(t)]$ and $[y_1(t), y_4(t)]$ are fundamental sets of solutions; $[y_2(t), y_3(t)]$ and $[y_4(t), y_5(t)]$ are not
29. Yes, $y = c_1e^{-x^2/2}\int_{x_0}^{x} e^{t^2/2}\,dt + c_2e^{-x^2/2}$
30. No
31. Yes, $y = \dfrac{1}{\mu(x)}\left[c_1\int_{x_0}^{x}\dfrac{\mu(t)}{t}\,dt + c_2\right], \quad \mu(x) = \exp\left[-\int\left(\dfrac{1}{x} + \dfrac{\cos x}{x}\right)dx\right]$
32. Yes, $y = c_1x^{-1} + c_2x$
34. $x^2\mu'' + 3x\mu' + (1 + x^2 - \nu^2)\mu = 0$
35. $(1 - x^2)\mu'' - 2x\mu' + \alpha(\alpha + 1)\mu = 0$
36. $\mu'' - x\mu = 0$
38. The Legendre and Airy equations are self-adjoint.

Section 3.3, page 158

1. Independent
2. Dependent
3. Independent
4. Dependent
5. Dependent
6. Independent
7. Independent if origin is interior to interval; otherwise dependent
8. Independent if origin is interior to interval; otherwise dependent
9. Independent; W is not always zero
10. Independent; W is not always zero
11. $W(c_1y_1, c_2y_2) = c_1c_2W(y_1, y_2) \neq 0$
12. $W(y_3, y_4) = -2W(y_1, y_2)$
13. $a_1b_2 - a_2b_1 \neq 0$
15. ct^2e^t
16. $c\cos t$
17. c/x
18. $c/(1 - x^2)$
20. $2/25$
21. $3\sqrt{e} \cong 4.946$
22. $p(t) = 0$ for all t
26. If t_0 is an inflection point, and $y = \phi(t)$ is a solution, then from the differential equation $p(t_0)\phi'(t_0) + q(t_0)\phi(t_0) = 0$.

Section 3.4, page 164

1. $e\cos 2 + ie\sin 2 \cong -1.1312 + 2.4717i$
2. $e^2\cos 3 - ie^2\sin 3 \cong -7.3151 - 1.0427i$
3. -1
4. $e^2\cos(\pi/2) - ie^2\sin(\pi/2) = -e^2i \cong -7.3891i$
5. $2\cos(\ln 2) - 2i\sin(\ln 2) \cong 1.5385 - 1.2779i$
6. $\pi^{-1}\cos(2\ln\pi) + i\pi^{-1}\sin(2\ln\pi) \cong -0.20957 + 0.23959i$

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7. $y = c_1 e^t \cos t + c_2 e^t \sin t$
8. $y = c_1 e^t \cos\sqrt{5}\,t + c_2 e^t \sin\sqrt{5}\,t$
9. $y = c_1 e^{2t} + c_2 e^{-4t}$
10. $y = c_1 e^{-t}\cos t + c_2 e^{-t}\sin t$
11. $y = c_1 e^{-3t}\cos 2t + c_2 e^{-3t}\sin 2t$
12. $y = c_1 \cos(3t/2) + c_2 \sin(3t/2)$
13. $y = c_1 e^{-t}\cos(t/2) + c_2 e^{-t}\sin(t/2)$
14. $y = c_1 e^{t/3} + c_2 e^{-4t/3}$
15. $y = c_1 e^{-t/2}\cos t + c_2 e^{-t/2}\sin t$
16. $y = c_1 e^{-2t}\cos(3t/2) + c_2 e^{-2t}\sin(3t/2)$
17. $y = \frac{1}{2}\sin 2t$; steady oscillation
18. $y = e^{-2t}\cos t + 2e^{-2t}\sin t$; decaying oscillation
19. $y = -e^{t-\pi/2}\sin 2t$; growing oscillation
20. $y = (1 + 2\sqrt{3})\cos t - (2 - \sqrt{3})\sin t$; steady oscillation
21. $y = 3e^{-t/2}\cos t + \frac{5}{2}e^{-t/2}\sin t$; decaying oscillation
22. $y = \sqrt{2}\,e^{-(t-\pi/4)}\cos t + \sqrt{2}\,e^{-(t-\pi/4)}\sin t$; decaying oscillation
23. (a) $u = 2e^{t/6}\cos(\sqrt{23}\,t/6) - (2/\sqrt{23})e^{t/6}\sin(\sqrt{23}\,t/6)$
    (b) $t = 10.7598$
24. (a) $u = 2e^{-t/5}\cos(\sqrt{34}\,t/5) + (7/\sqrt{34})\,e^{-t/5}\sin(\sqrt{34}\,t/5)$
    (b) $T = 14.5115$
25. (a) $y = 2e^{-t}\cos\sqrt{5}\,t + [(\alpha + 2)/\sqrt{5}]\,e^{-t}\sin\sqrt{5}\,t$
    (b) $\alpha = 1.50878$
    (c) $t = \{\pi - \arctan[2\sqrt{5}/(2+\alpha)]\}/\sqrt{5}$
    (d) $\pi/\sqrt{5}$
26. (a) $y = e^{-at}\cos t + ae^{-at}\sin t$
    (b) $T = 1.8763$
    (c) $\alpha = \frac{1}{4}$, $T = 7.4284$; $\alpha = \frac{1}{2}$, $T = 4.3003$; $\alpha = 2$, $T = 1.5116$
35. Yes, $y = c_1\cos x + c_2 \sin x$, $x = \int e^{-t^2/2}\,dt$
36. No
37. Yes, $y = c_1 e^{-t^2/4}\cos(\sqrt{3}\,t^2/4) + c_2 e^{-t^2/4}\sin(\sqrt{3}\,t^2/4)$
39. $y = c_1\cos(\ln t) + c_2\sin(\ln t)$
40. $y = c_1 t^{-1} + c_2 t^{-2}$
41. $y = c_1 t^{-1}\cos(\frac{1}{2}\ln t) + c_2 t^{-1}\sin(\frac{1}{2}\ln t)$
42. $y = c_1 t^6 + c_2 t^{-1}$

Section 3.5, page 172

1. $y = c_1 e^t + c_2 te^t$
2. $y = c_1 e^{-t/3} + c_2 te^{-t/3}$
3. $y = c_1 e^{-t/2} + c_2 e^{3t/2}$
4. $y = c_1 e^{-3t/2} + c_2 te^{-3t/2}$
5. $y = c_1 e^t\cos 3t + c_2 e^t \sin 3t$
6. $y = c_1 e^{3t} + c_2 te^{3t}$
7. $y = c_1 e^{-t/4} + c_2 e^{-4t}$
8. $y = c_1 e^{-3t/4} + c_2 te^{-3t/4}$
9. $y = c_1 e^{2t/5} + c_2 te^{2t/5}$
10. $y = e^{-t/2}\cos(t/2) + c_2 e^{-t/2}\sin(t/2)$
11. $y = 2e^{2t/3} - \frac{7}{3}te^{2t/3}$, $y \to -\infty$ as $t \to \infty$
12. $y = 2te^{3t}$, $y \to \infty$ as $t \to \infty$
13. $y = -e^{-t/3}\cos 3t + \frac{5}{9}e^{-t/3}\sin 3t$, $y \to 0$ as $t \to \infty$
14. $y = 7e^{-2(t+1)} + 5te^{-2(t+1)}$, $y \to 0$ as $t \to \infty$
15. (a) $y = e^{-3t/2} - \frac{5}{2}te^{-3t/2}$ (b) $t = \frac{2}{5}$
    (c) $t_0 = 16/15$, $y_0 = -\frac{5}{3}e^{-8/5} \cong -0.33649$
    (d) $y = e^{-3t/2} + (b + \frac{3}{2})te^{-3t/2}$; $b = -\frac{3}{2}$
16. $y = 2e^{t/2} + (b - 1)te^{t/2}$; $b = 1$
17. (a) $y = e^{-t/2} + \frac{5}{2}te^{-t/2}$ (b) $t_M = \frac{8}{5}$, $y_M = 5e^{-4/5} \cong 2.24664$
    (c) $y = e^{-t/2} + (b + \frac{1}{2})te^{-t/2}$
    (d) $t_M = 4b/(1 + 2b) \to 2$ as $b \to \infty$;
    $y_M = (1 + 2b)\exp[-2b/(1 + 2b)] \to \infty$ as $b \to \infty$
18. (a) $y = ae^{-2t/3} + (\frac{2}{3}a - 1)te^{-2t/3}$ (b) $a = \frac{3}{2}$
23. $y_2(t) = t^3$
24. $y_2(t) = t^{-2}$
25. $y_2(t) = t^{-1}\ln t$
26. $y_2(t) = te^t$
27. $y_2(x) = \cos x^2$
28. $y_2(x) = x$