35. (a) $v' - [x(t) + b]v = b$

   (b) $v = \left[b \int \mu(t)\,dt + c\right]\Big/\mu(t), \quad \mu(t) = \exp[-(at^2/2) - bt]$

36. $y = c_1 t^{-1} + c_2 + \ln t$

37. $y = c_1 \ln t + c_2 + t$

38. $y = (1/k)\ln|(k-t)/(k+t)| + c_2$ if $c_1 = k^2 > 0$;

   $y = (2/k)\arctan(t/k) + c_2$ if $c_1 = -k^2 < 0$; $y = -2t^{-1} + c_2$ if $c_1 = 0$;   also $y = c$

39. $y = \pm\frac{2}{3}(t - 2c_1)\sqrt{t + c_1} + c_2$;   also $y = c$ *Hint:* $\mu(v) = v^{-3}$ is an integrating factor.

40. $y = c_1 e^{-t} + c_2 - te^{-t}$

41. $c_1^2 y = c_1 t - \ln|1 + c_1 t| + c_2$ if $c_1 \neq 0$; $y = \frac{1}{2}t^2 + c_2$ if $c_1 = 0$;   also $y = c$

42. $y^2 = c_1 t + c_2$

43. $y = c_1 \sin(t + c_2) = k_1 \sin t + k_2 \cos t$

44. $\frac{1}{3}y^3 - 2c_1 y + c_2 = 2t$;   also $y = c$

45. $t + c_2 = \pm\frac{2}{3}(y - 2c_1)(y + c_1)^{1/2}$

46. $y \ln|y| - y + c_1 y + t = c_2$;   also $y = c$

47. $e^y = (t + c_2)^2 + c_1$

48. $y = \frac{4}{3}(t + 1)^{3/2} - \frac{1}{3}$

49. $y = 2(1 - t)^{-2}$

50. $y = 3\ln t - \frac{3}{2}\ln(t^2 + 1) - 5\arctan t + 2 + \frac{3}{2}\ln 2 + \frac{5}{4}\pi$

51. $y = \frac{1}{2}t^2 + \frac{3}{2}$

CHAPTER 3

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1. $y = c_1 e^t + c_2 e^{-3t}$

2. $y = c_1 e^{-t} + c_2 e^{-2t}$

3. $y = c_1 e^{t/2} + c_2 e^{-t/3}$

4. $y = c_1 e^{t/2} + c_2 e^t$

5. $y = c_1 + c_2 e^{-5t}$

6. $y = c_1 e^{3t/2} + c_2 e^{-3t/2}$

7. $y = c_1 \exp[(9 + 3\sqrt{5})t/2] + c_2 \exp[(9 - 3\sqrt{5})t/2]$

8. $y = c_1 \exp[(1 + \sqrt{3})t] + c_2 \exp[(1 - \sqrt{3})t]$

9. $y = e^t; \quad y \to \infty$ as $t \to \infty$

10. $y = \frac{5}{2}e^{-t} - \frac{1}{2}e^{-3t}; \quad y \to 0$ as $t \to \infty$

11. $y = 12e^{t/3} - 8e^{t/2}; \quad y \to -\infty$ as $t \to \infty$

12. $y = -1 - e^{-3t}; \quad y \to -1$ as $t \to \infty$

13. $y = \frac{1}{26}(13 + 5\sqrt{13})\exp[(-5 + \sqrt{13})t/2] + \frac{1}{26}(13 - 5\sqrt{13})\exp[(-5 - \sqrt{13})t/2];$

   $y \to 0$ as $t \to \infty$

14. $y = (2/\sqrt{33})\exp[(-1 + \sqrt{33})t/4] - (2/\sqrt{33})\exp[(-1 - \sqrt{33})t/4]; \quad y \to \infty$ as $t \to \infty$

15. $y = \frac{1}{10}e^{-9(t-1)} + \frac{9}{10}e^{t-1}; \quad y \to \infty$ as $t \to \infty$

16. $y = -\frac{1}{2}e^{(t+2)/2} + \frac{3}{2}e^{-(t+2)/2}; \quad y \to -\infty$ as $t \to \infty$

17. $y'' + y' - 6y = 0$

18. $2y'' + 5y' + 2y = 0$

19. $y = \frac{1}{4}e^t + e^{-t}$; minimum is $y = 1$ at $t = \ln 2$

20. $y = -e^t + 3e^{t/2}$; maximum is $y = \frac{9}{4}$ at $t = \ln(9/4)$, $y = 0$ at $t = \ln 9$

21. $\alpha = -2$

22. $\beta = -1$

23. $y \to 0$ for $\alpha < 0$; y becomes unbounded for $\alpha > 1$

24. $y \to 0$ for $\alpha < 1$; there is no α for which all nonzero solutions become unbounded

25. (a) $y = \frac{1}{5}(1 + 2\beta)e^{-2t} + \frac{1}{5}(4 - 2\beta)e^{t/2}$ (b) $y \cong 0.71548$ when $t = \frac{2}{5}\ln 6 \cong 0.71670$

   (c) $\beta = 2$

26. (a) $y = (6 + \beta)e^{-2t} - (4 + \beta)e^{-3t}$

   (b) $t_m = \ln[(12 + 3\beta)/(12 + 2\beta)], \quad y_m = \frac{4}{27}(6 + \beta)^3/(4 + \beta)^2$

   (c) $\beta = 6(1 + \sqrt{3}) \cong 16.3923$

   (d) $t_m \to \ln(3/2), \quad y_m \to \infty$

27. (a) $y = d/c$

   (b) $aY'' + bY' + cY = 0$

28. (a) $b > 0$ and $0 < c < b^2/4a$

   (b) $c < 0$

   (c) $b < 0$ and $0 < c < b^2/4a$