Math 347

Due Thursday, November 1, 2007.

Note: You will need the result from Problem 2 to complete Discrete Dynamical Systems Homework 3.

Solutions of linear and affine dynamical systems

Definition. The **general solution** to a first order dynamical system

$$A(n+1) = f(A(n)),\ n = 0\ldots$$

is a function $A(n)$ with domain $n = 0, \ldots$, which

- satisfies the dynamical system when substituted in for $A(n)$ and $A(n+1)$, and
- involves a constant c which can be determined once an initial value is given.

Definition. The **particular solution** to a first order dynamical system

$$A(n+1) = f(A(n)),\ n = 0\ldots$$

with an initial value given, say $A(0) = a_0$ is a function $A(n)$ with domain $n = 0, \ldots$, which

- satisfies the dynamical system when substituted in for $A(n)$ and $A(n+1)$, and
- when $n = 0$, satisfies the equation $A(0) = a_0$.

In general, it is very difficult (or impossible) to find closed-form solutions for dynamical systems that arise in mathematical modeling, and we thus use graphical and numerical techniques to analyze dynamical systems (as we have done in class so far). However, we can find solutions to linear and affine dynamical systems. Additionally, when we model situations, we can often make simplifying assumptions that lead to linear dynamical systems, and we can use the solutions of linear systems to develop a better understanding of the situation we are modeling. We can then try to make our model more realistic by adding secondary assumptions.

1. Consider the first order linear dynamical system

$$A(n+1) = rA(n),\ n = 0, \ldots.$$

Show that the general solution for the system is

$$A(n) = cr^n,$$

and that the particular solution given the initial value $A(0) = a_0$ is

$$A(k) = a_0 r^k.$$

2. Find the general and particular solutions for the first order affine dynamical system

$$A(n+1) = rA(n) + b.$$

Hint: Make the substitution $A(k) = E(k) + a$, where a is the equilibrium point for the dynamical system $A(n+1) = rA(n)+b$, and find the general solution for the system $E(n+1) = f(E(n))$ (of course, to do this, you will need to determine the function f).